imagine mypath®

FOUNDATIONS PAPER

How Imagine MyPath® K–12 Aligns With Research on

Effective Reading and Mathematics Instruction

The Challenge

Educators across the country have amplified their efforts to improve student achievement after the National Assessment of Educational Progress (NAEP) revealed that 35% of fourth-grade students scored Proficient in reading and 41% scored Proficient in mathematics (2019a, 2019b). Similarly, 34% of eighth-grade students scored Proficient in reading and 34% scored Proficient in mathematics (NAEP, 2019a, 2019b). The difficulty lies in the fact that classrooms are becoming increasingly academically diverse, with children exhibiting different abilities, interests, and learning needs (Subban, 2006). Consider the following third-grade classroom (Figure 1): In reading, Nia struggles with early literacy skills and Diego comprehends text above grade level. In mathematics, Tali has difficulty with whole-number concepts, while Jordan struggles with measurement and data concepts. As can be seen in this example, teaching is a constant balancing act, and differentiating instruction for a classroom of academically diverse students is challenging.

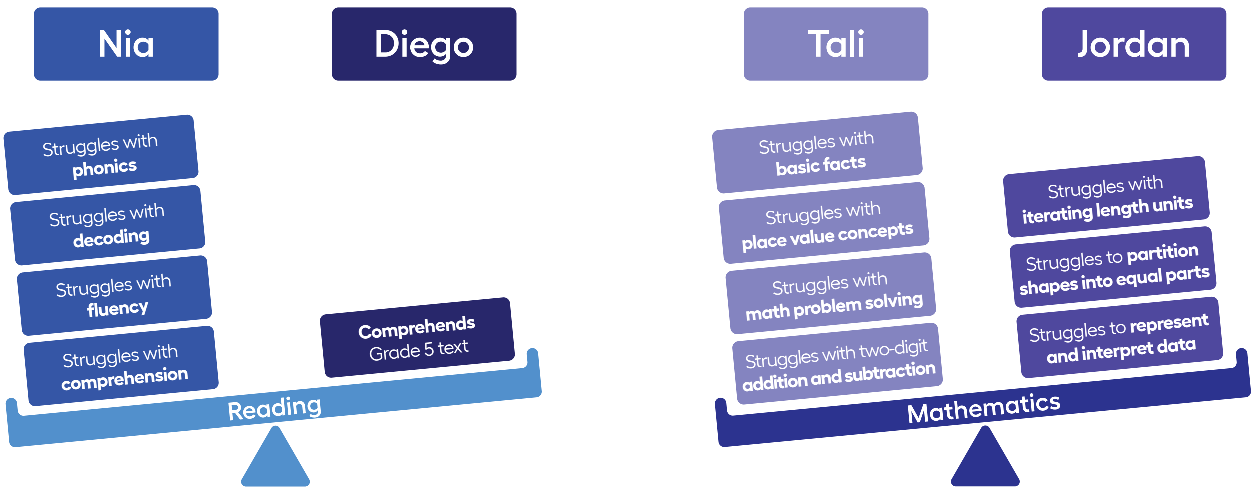


Figure 1. *Example of an academically diverse classroom.*

A growing number of educators have turned to technology to provide personalized interventions that meet the unique learning needs of students in their classrooms (Shemshack et al., 2021). Personalized learning is an approach to teaching that adapts instruction based on students' strengths and areas of growth, rather than focusing solely on grade-level content. The objective is to optimize student learning by meeting students where they are academically. Research shows that blending technology with teacher-led instruction is especially effective in helping struggling students succeed (Cavanaugh et al., 2013; Darling-Hammond et al., 2014; Fazal & Bryant, 2019; Means et al., 2013; Pane et al., 2015).

The Solution

Imagine MyPath is a student-centered reading and mathematics program designed to close achievement gaps and maximize academic growth for students in Grades K–12. Imagine MyPath is a new supplemental curriculum that utilizes Smart Sequencer™ technology to prioritize essential skills and create individual learning paths (ILPs) in reading and mathematics. ILPs are grounded in research, and continuously adapt to ensure success among academically diverse learners. All Imagine MyPath lessons are age appropriate and intentionally designed to align with each student's grade and skill level. A student's chronological grade determines their experience in the program and the presentation of information, but their skill level determines the types of questions presented.

Imagine MyPath:

- assesses students' abilities and accurately identifies their instructional grade level;
- prioritizes essential reading and mathematics skills to optimize grade-level learning;
- provides a sequence of age-appropriate, high-impact lessons that continuously adapt to students' strengths and areas of growth;
- provides ongoing data and analytics for educators;
- incorporates offline downloadable teaching resources for students in need of additional support; and
- offers engaging interactive rewards to maintain student engagement and motivate them to persevere and work hard.

Imagine MyPath prioritizes the following instructional domains and the essential skills within them to help students accelerate toward grade-level achievement.

In reading, instruction focuses on:

- phonics
- vocabulary
- fluency
- reading comprehension

In mathematics, instruction focuses on:

- number and operations
- algebra
- measurement and data
- geometry

The curriculum uniquely leverages a cycle of assessment, assignment, adaptivity, analysis, and action to create an ILP that delivers an adaptive sequence of lessons so students efficiently catch up, keep up, and get ahead (Figure 2).

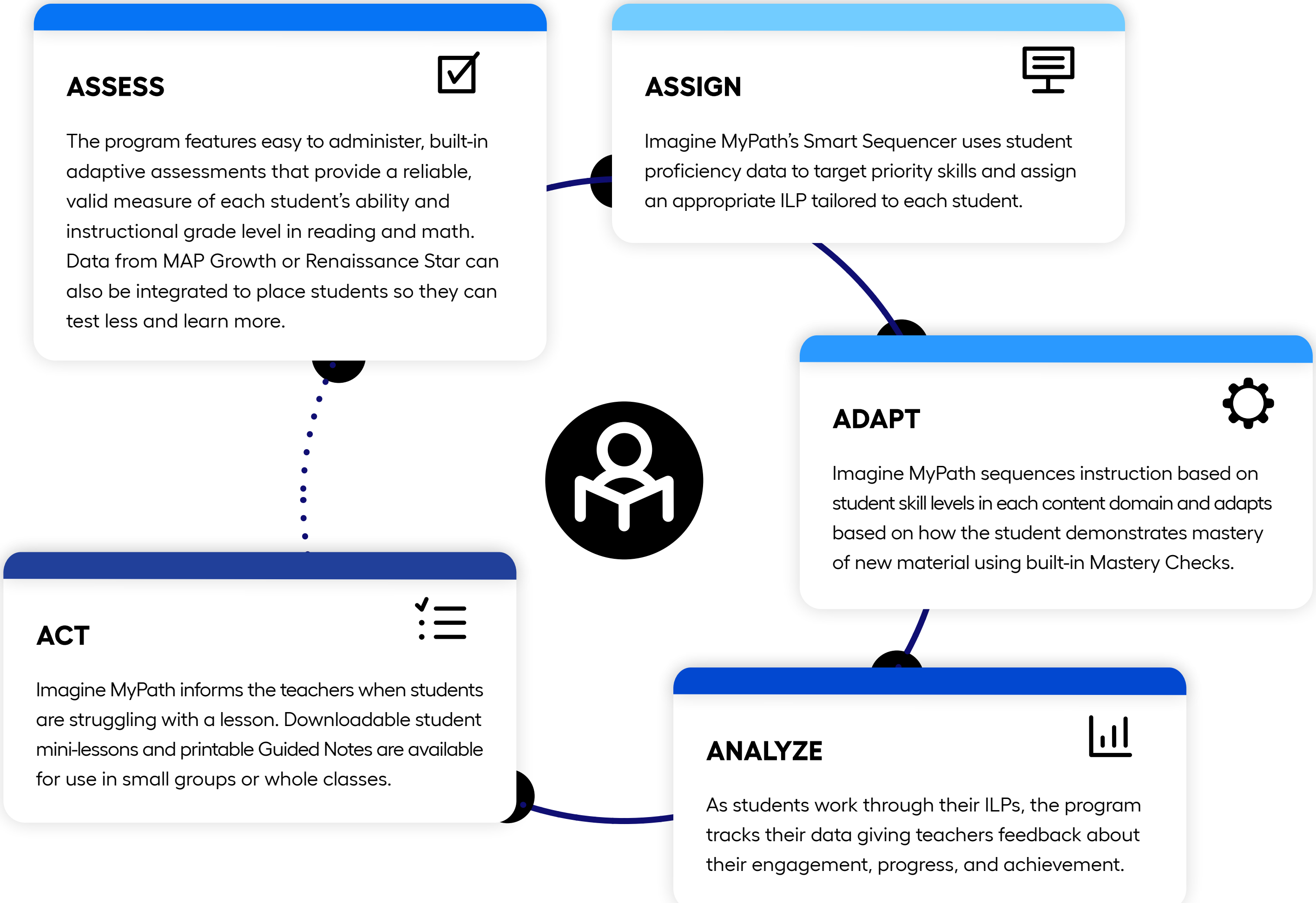


Figure 2. *Imagine MyPath instructional cycle.*

Imagine MyPath Learning Experience

Student **assessment** data (e.g., Imagine MyPath Assessment or third-party data from NWEA Measures of Academic Progress [MAP]® Growth™ or Renaissance Star®) provides a measure of each student's performance and instructional grade level in reading and mathematics. Then, Imagine MyPath's Smart Sequencer™ automatically **assigns** ILPs based on the student's overall scaled score or instructional grade level for the assessment, as well as their chronological grade. The instructional sequence seamlessly **adapts** in response to student performance. Integrated Mastery Checks drive the curriculum's adaptivity within lessons and across students' ILPs, targeting skill gaps as they emerge and allowing students to skip lessons on concepts they have already mastered. For each concept, students begin by taking a five-item assessment. If they demonstrate mastery, they skip the lesson and move to the next skill in their ILP. If they struggle, the curriculum provides age-appropriate lessons designed to get them back on track. As students work through their ILP, the program collects performance data within an intuitive dashboard. Teacher dashboards allow teachers to **analyze** student data by domain in real time, equipping them with actionable insights into students' strengths and areas of growth. Teachers have access to downloadable offline resources to quickly **act** if students demonstrate consistent difficulty with a particular concept. Students' performance on these tasks helps teachers plan more intensive interventions, if necessary. This circular pattern continues, propelling students toward mastery of grade-level content.

Imagine MyPath Logic Model

The logic model below provides a conceptual model of how Imagine MyPath is intended to work, the resources required to make it effective, and the outcomes that teachers can expect students to demonstrate (Figure 3). The program inputs and classroom activities list the planned work that is needed to successfully launch Imagine MyPath and generate the outputs that lead to the short- and long-term outcomes.

Program Inputs

IMAGINE MYPATH

- Explicit, adaptive, and personalized reading and math instruction
- Smart Sequencer targets learning gaps and pinpoints prerequisite skills needed to master grade-level standards
- Embedded MyPath Assessment establishes students' individualized learning paths and measures growth
- NWEA MAP Growth and Renaissance Star integrations available to establish students' individualized learning paths without additional testing
- Instruction focused on essential grade-level concepts and skills to accelerate learning
- Multi-sensory and game-based motivation system
- Age-appropriate content, regardless of skill level
- Interactive and graphic scaffolds to support diverse learners, including hyperlinked vocabulary words, metacognitive bubbles, calculator and graphing tools, and graphic organizers
- Sensory scaffolds provided with multimedia support that combine video, audio, text, and narration, as well as interactive manipulatives
- On-screen translation available for over 60 languages, text-to-speech read-aloud for over 45 languages, and
- K–5 math lessons available in Spanish
- Data dashboard delivers actionable data to teachers to inform instructional decision-making
- Assignment Builder allows teachers to explore, preview and assign lessons and formative assessments

IMAGINE LEARNING

- Onboarding and implementation support
- Professional development and coaching for teachers and administrators
- Flexible implementation models for content delivery
- Customer support to troubleshoot issues

DISTRICT

- Technology: networked computers or mobile devices, headphones, and supporting hardware and software
- Enable appropriate language support for students who may require it
- School and district infrastructure to support technology use
- Teacher buy-in and readiness to adopt technology
- School implementation plan

Classroom Activities

STUDENT ACTIVITIES

- Students at grade level use Imagine MyPath for 30–60 minutes per subject per week
- Students below grade level use Imagine MyPath for 60–90 minutes per subject per week
- Students spend 15-20 minutes per session working in the program
- Students complete offline activities if and when assigned by teacher

TEACHER ACTIVITIES

- Teachers spend approximately 30 minutes per week reviewing program data through the Imagine MyPath data dashboard and planning instruction to meet student needs
- Teachers ensure all students are meeting minimum usage and progress goals
- Teachers identify and act upon opportunities for small- or whole-group follow-up based on lesson performance
- Teachers use offline resources to provide additional practice or support when necessary
- Teachers manually assign learning paths or activities to students when appropriate

Outputs

STUDENT OUTPUTS

- Students placed at or above grade level completed at least one lesson per subject per week
- Students placed below grade level completed at least two lessons per subject per week
- Students demonstrated content engagement based on progress in the program
- Students demonstrated content mastery based on lessons passed in the program

TEACHER OUTPUTS

- Teachers completed professional development and felt prepared to implement Imagine MyPath
- Teachers built their understanding of students' strengths and areas of growth
- Teachers made informed calibrations of student educational pathways based on student performance in Imagine MyPath
- Teachers provided small- or whole-group support to students based on performance in Imagine MyPath

Outcomes

SHORT-TERM

- Increased student engagement in math and reading
- Increased math and reading content mastery
- Increased performance on standardized reading and math formative and summative assessments
- Students experience success in math and reading content at their grade level

LONG-TERM

- Students are prepared to receive grade-level instruction in later grades
- Students increase reading and math achievement on state or nationally normed assessments

Figure 3. *Imagine MyPath logic model.*

Research-Based Solution

Imagine MyPath incorporates six research-based principles of effective teaching that accelerate reading and mathematics achievement.

1. Prioritize content and adapt learning based on student performance.
2. Provide accessible, explicit, and scaffolded instruction to ensure success among academically diverse learners.
3. Incorporate evidence-based practices for teaching reading.
4. Incorporate evidence-based practices for teaching mathematics.
5. Deliver actionable data to inform instructional decision making.
6. Optimize student motivation and engagement.

Principle 1. Prioritize Content and Adapt Learning Based On Student Performance

When educators **prioritize reading and mathematics skills**, they help students develop a deep understanding of essential grade-level skills. Researchers agree that emphasizing the depth of learning, rather than the breadth, is more effective than covering each concept or skill within the standards (Ainsworth, 2013). According to the Council of Great City Schools (2020), “Prioritizing content and learning does not mean that students will be deprived of critical knowledge, or that their education will be any less diverse or rich” (p. 5). Rather, instruction should systematically address areas of unfinished or interrupted learning in the context of grade-level standards, foster connections between prior knowledge and new content, and help students develop the skills needed to understand the topic conceptually (Gersten, Beckmann, et al., 2009). For example, Kim et al. (2021) and Fuchs et al. (2021) tested the efficacy of these principles in a reading and mathematics intervention that prioritized domain-specific grade-level standards, and developed a logical sequence of high-impact skills designed to scale students to success. In both studies, intervention students outperformed equivalent controls on a range of measures. These findings show how addressing essential grade-level skills enables educators to accelerate learning and help address unfinished learning.

Prioritization is critical, but insufficient for efficiently driving student success. Because students have diverse experiences and knowledge, research supports continuously adapting instruction to address students’ unique learning needs (Ankrum et al., 2020; Athanases et al., 2015; Clark & Mayer, 2016; Tomlinson, 2014). Modifying the content and presentation of material—known as **adaptive teaching**—is critical for promoting a deeper transfer of learning (Fuchs et al., 2017; Parsons & Vaughn, 2016; Vagle, 2016) and meeting the ongoing challenge of inclusive teaching (Westwood, 2018). Nearly 15 years of research suggest that students who receive adaptive instruction demonstrate significantly greater gains in reading and mathematics than those who receive nonadaptive methods of instruction (Aleven et al., 2017; Alshammari et al., 2016; Ma et al., 2014; VanLehn, 2011; Ysseldyke & Tardrew, 2007). Adaptive learning models provide age-appropriate, dynamic, efficient, and engaging instruction and are considered more effective than instructional models known to emphasize rote repetition of skills (Linn et al., 2000; Salinger, 2003). This is especially true for older students struggling with skills covered in lower grade levels. Adaptive teaching can streamline student success by focusing on grade-level content, essential skills, and students’ strengths.

How Imagine MyPath Prioritizes Content and Adapts Learning

Imagine MyPath prioritizes grade-level content and essential skills to accelerate students’ ability to comprehend text and develop a conceptual understanding of mathematics. Program designers use coherence mapping (Student Achievement Partners, 2020) and draw on research to determine the most essential skills. Coherence mapping refers to the underlying idea that concepts across reading and mathematics domains connect within and across grades. This coherence map integrates with Smart Sequencer™ technology so that each student’s ILP addresses learning gaps and targets prerequisite skills needed to master grade-level standards.

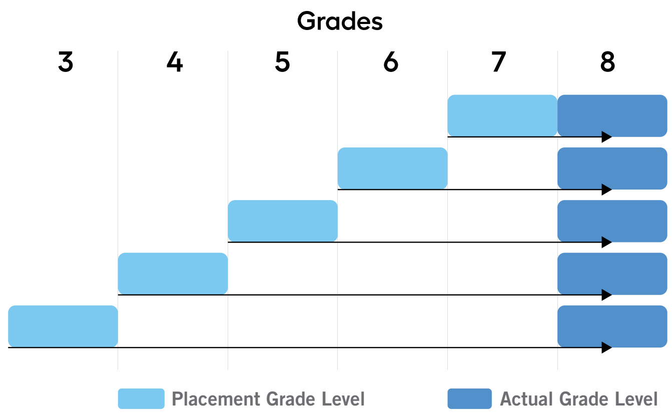


Figure 4. *Imagine MyPath prioritizes essential skills.*

For instance, if a student performs significantly below grade level, they likely demonstrate difficulty with a broader range of concepts. In that case, the ILP increases learning efficiency by organizing essential skills into progressions (predetermined, research-based, and purposeful sets of skills, agnostic of grade level) to accelerate growth (Figure 4). These progressions allow students to comprehensively focus on fewer skills to guide their ILP toward grade-level content.

In reading, Imagine MyPath prioritizes comprehension of literary and informational texts. All Grades K–2 students working on or slightly below grade level receive explicit instruction on reading foundations. Early literacy lessons take a strategy-based approach by emphasizing the process behind each skill through a select number of examples and practice items. This structure prepares students for their assigned ILP, while also serving the program goal of accelerating achievement. Students performing on or slightly below grade level in Grades 3–5 immediately begin with comprehension lessons commensurate with their placement level. Students in Grades 3–5 who place at least two grade levels behind begin their ILP with an *Early Literacy Bundle* to review essential skills that support progress toward grade-level comprehension. Students in Grades 6–8 who place at least two grade levels behind and students in Grades 9–12 who place into Grades K–6 will also begin their ILP with an *Early Literacy Bundle*. This allows students to rebuild foundational skills needed to read grade-level texts.

There are six versions of the *Early Literacy Bundle* (Figure 5). Each accelerator bundle includes four lessons focused on phonics, vocabulary, and fluency, with varying complexity based on students' actual grade (which affects presentation style) and placement grade (which affects difficulty level). However, *Bundle 6* only includes two lessons focused on vocabulary.

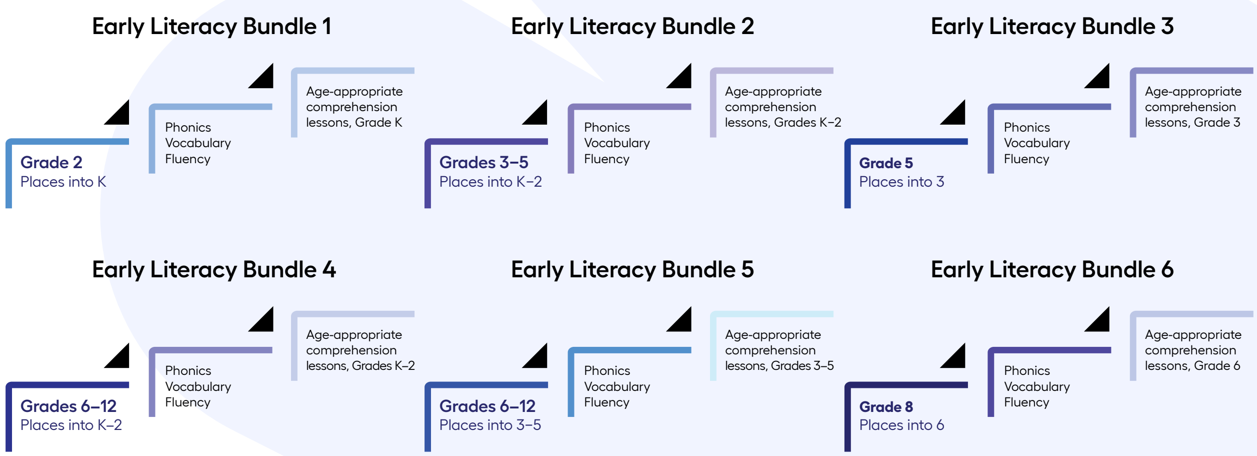


Figure 5. *Imagine MyPath Early Literacy Bundles.*

In mathematics, prioritization is domain focused. Imagine MyPath includes lessons across all domains—number and operations, algebra, measurement and data, and geometry. The program incorporates rigorous mathematics standards and practices to maximize students' conceptual understanding of grade-level concepts. Because it prioritizes key lessons, students do not need to receive all lessons within each domain. Instead, the curriculum emphasizes the most essential skills to efficiently scale students up to grade level. This helps students build a foundational understanding before moving on to more sophisticated concepts.

To provide a more granular view of how a student's ILP prioritizes mathematics content within a specific grade-level skill, consider a Grade 9 student who is performing three grade levels below in algebra (Figure 6). This student struggles to solve problems involving linear equations. Therefore, the student's ILP reviews essential skills required to master the linear equations (e.g., solving one-step equations, solving problems involving rational numbers, solving with variables on both sides). The progressions become more refined and efficient, which accelerates growth to grade-level proficiency.

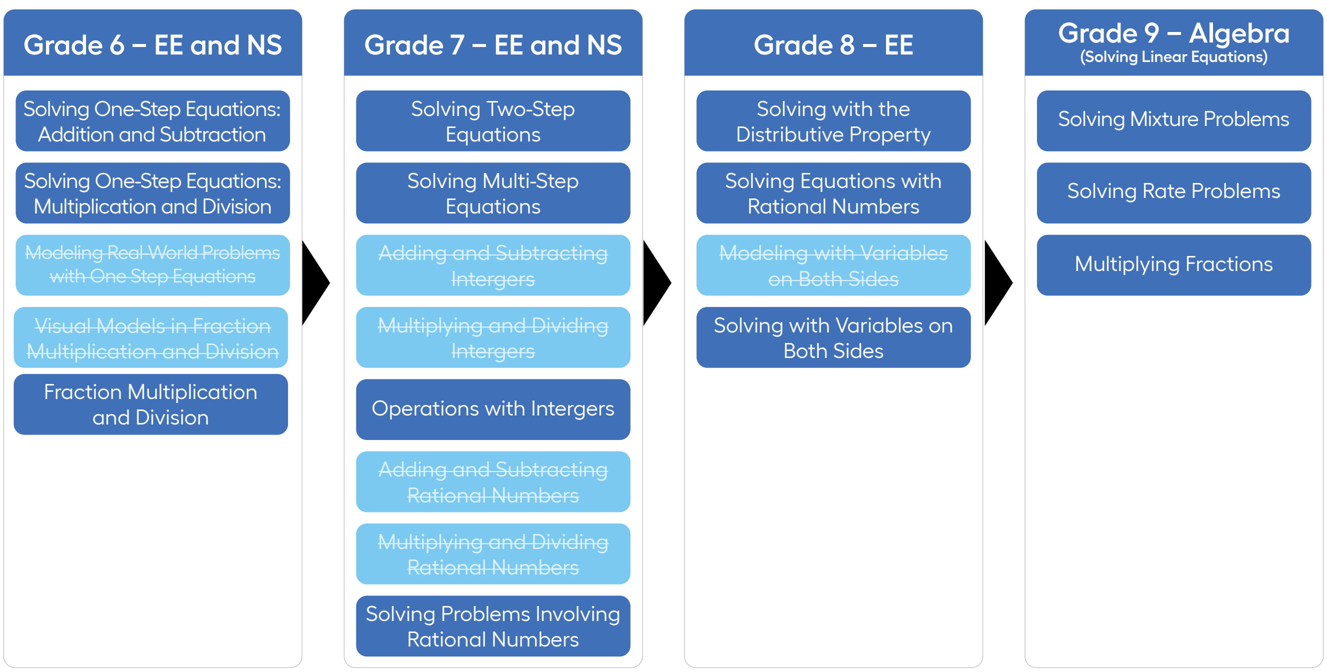


Figure 6. *Student's individual learning path.*

Each student's current level of understanding (based on assessment data) determines the starting point for their ILP. The graphic below shows how Imagine MyPath K–5 adapts to students' knowledge *within* a lesson and the general instructional framework (note that the reading foundations lessons do not follow this exact activity guide) (Figure 7). In the K–5 environment, each lesson begins with a five-item Mastery Check. If a student answers four or more items correctly (or 80%), they test out of that lesson and progress to the next. If not, they receive Instruction that integrates relevant examples, real-world connections, and modeling to build their understanding of the concept. Students have a second chance to demonstrate proficiency by taking another Mastery Check, which contains different items and response options. If they pass, they move on to the next lesson. If not, they receive Supported and Independent Practice. If a student passes the third Mastery Check, they move on. If not, the teacher receives a notification that the student requires additional support. Imagine MyPath offers downloadable offline resources that enable teachers to reteach the concept or skill (Figure 8). Students move on to the next lesson in their ILP once the teacher determines they have mastered the skill.

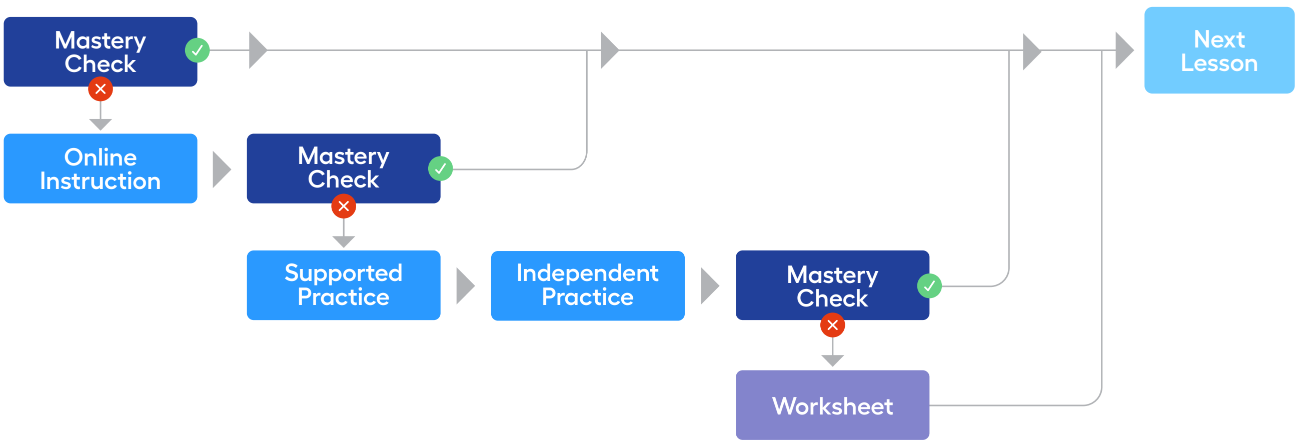
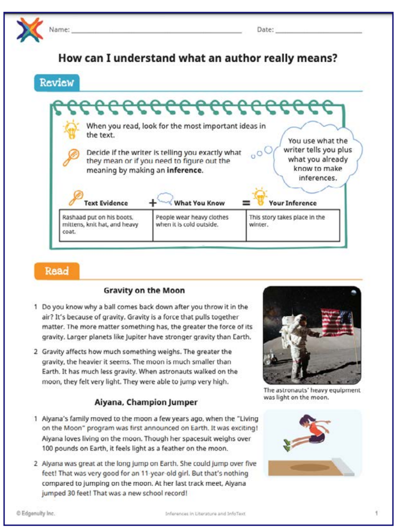


Figure 7. *Imagine MyPath Grades K–5 lesson structure.*



Name: ______ Date: ______

How can I understand what an author really means?

Review

When you read, look for the most important ideas in the text.

Decide if the writer is telling you exactly what they mean or if you need to figure out the meaning by making an **inference**.

You use what the writer tells you plus what you already know to make inferences.

Text Evidence +	What You Know =	Your Inference
Rashaad put on his boots, mittens, knit hat, and heavy coat.	People wear heavy clothes when it is cold outside.	This story takes place in the winter.

Read

Gravity on the Moon

1 Do you know why a ball comes back down after you throw it in the air? It's because of gravity. Gravity is a force that pulls together matter. The more matter something has, the greater the force of its gravity. Larger planets like Jupiter have stronger gravity than Earth.

2 Gravity affects how much something weighs. The greater the gravity, the heavier it seems. The moon is much smaller than Earth. It has much less gravity. When astronauts walked on the moon, they felt very light. They were able to jump very high.

The astronauts' heavy equipment was light on the moon.

Aiyana, Champion Jumper

1 Aiyana's family moved to the moon a few years ago, when the "Living on the Moon" program was first announced on Earth. It was exciting! Aiyana loves living on the moon. Though her spacesuit weighs over 100 pounds on Earth, it feels light as a feather on the moon.

2 Aiyana was great at the long jump on Earth. She could jump over five feet! That was very good for an 11-year-old girl. But that's nothing compared to jumping on the moon. At her last track meet, Aiyana jumped 30 feet! That was a new school record!

© Edgenuity Inc. Inferences in Literature and InfoText 1

Figure 8. *Imagine MyPath Grades K–5 downloadable resource.*

Imagine MyPath 6–12 also adapts to students' knowledge within a lesson. Figure 9 displays the general instructional framework for this learning environment. Each lesson begins with a five-item Mastery Check. If the student answers four or more items correctly (or 80%), they test out of that lesson and move on to the next. If they do not meet the passing threshold, students receive Instruction. Students have access to Guided Notes, which can be used throughout the lesson (Figure 10). These can be printed by the teacher or from the students' dashboard. After Instruction, students take a second Mastery Check. If students demonstrate proficiency, they move on to the next lesson. If not, they receive additional Instruction and Practice. This reinforces the concept and allows students to apply what they have learned. If the student fails the third and final Mastery Check, the teacher is notified. Again, the teacher can download offline resources that accompany each lesson to provide individualized support before the student moves on to the next lesson (Figure 11).

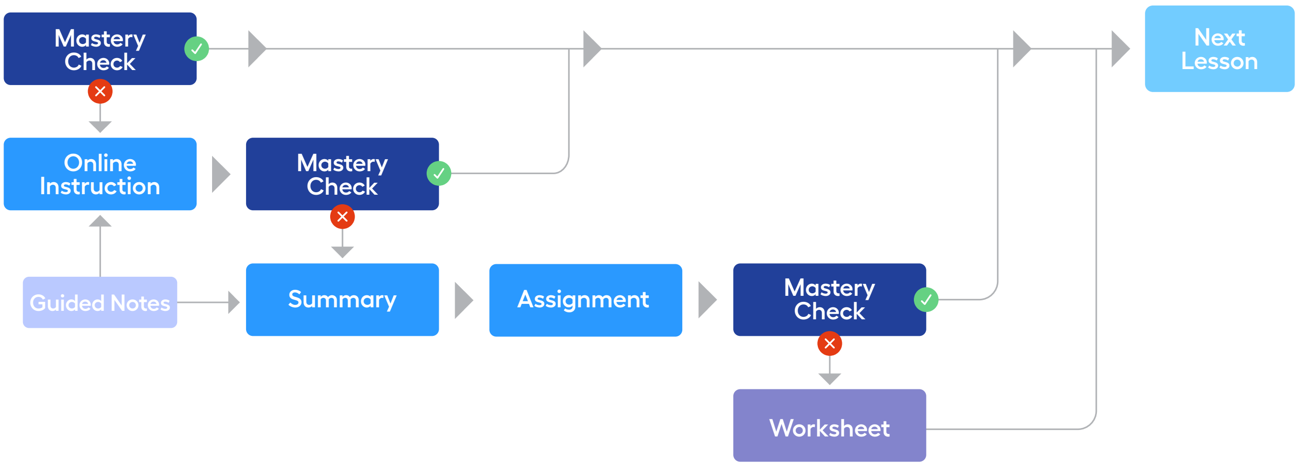


Figure 9. *Imagine MyPath Grades 6–12 lesson structure.*

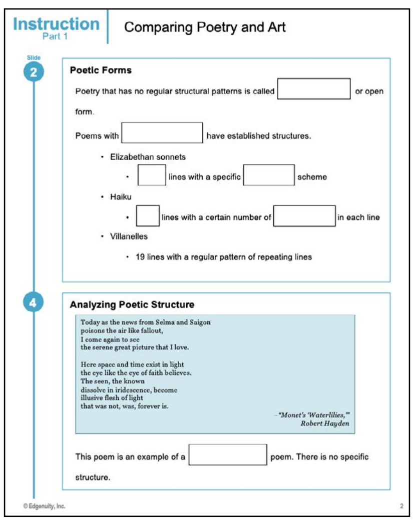


Figure 10. *Guided Notes for Grades 6–12 students.*

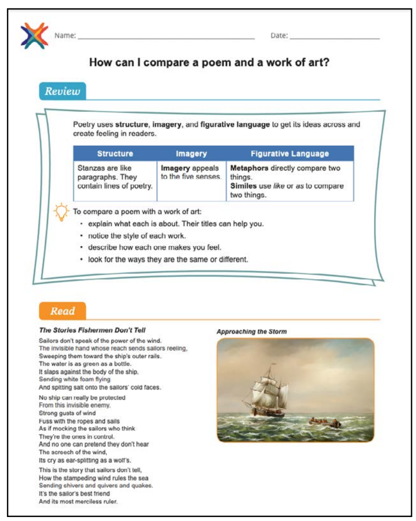


Figure 11. *Imagine MyPath Grades 6–12 downloadable resource.*

Imagine MyPath's Smart Sequencer™ also adapts the curriculum *across* lessons. If students display sustained and unproductive struggle, their ILP recalibrates to the previous grade level to provide instruction that allows them to get back on track. Teachers can adjust ILPs to include foundational lessons within and across grade levels to reinforce skills, if necessary. Teachers can also customize the lessons (i.e., add lessons below or above grade level) to augment the instruction students receive. These changes are automatically reflected in the student's ILP the next time they log in to the program.

A notable feature of Imagine MyPath is its unique ability to provide instruction on the same skill to students across Grades K–12. However, the presentation of the skill reflects the students' chronological age. Students in the upper grade levels who require skills from lower grade levels receive modified age-appropriate material (e.g., changes in the graphical presentation, organization, storytelling, context orientation) to be more relevant to their developmental age. For example, students in Grades 6–12 who require lessons focused on Grades 3–5 skills receive a presentation style commensurate with their maturity level.

Figures 12 and 13 both emphasize the same mathematics skill, dividing by a unit fraction. However, the presentation style of Figure 12 is designed for students in Grades K–5, whereas Figure 13 is modified for students in Grades 6–12. In this example, the visual models, representations, vocabulary, strategies, colors, and overall layout reflect the students' respective grade levels. Similarly, Figures 14 and 15 display a lesson focused on cause and effect. The presentation style of Figure 14 is designed for students in Grades K–5, while Figure 15 is modified for students in Grades 6–12.

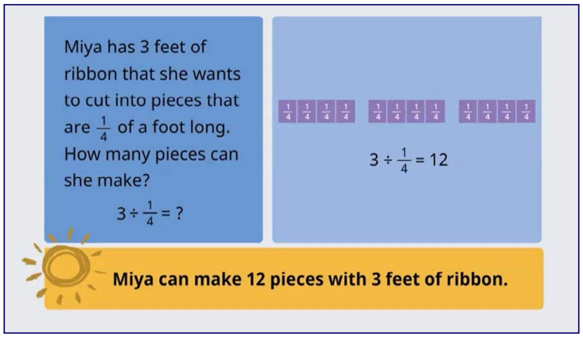


Figure 12. *Mathematics lesson designed for Grades K–5 students.*

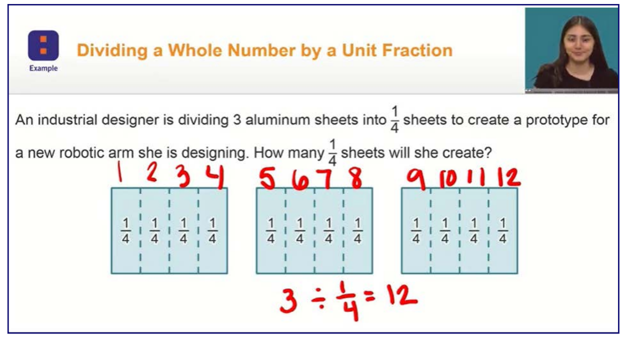


Figure 13. *Mathematics lesson designed for Grades 6–12 students.*

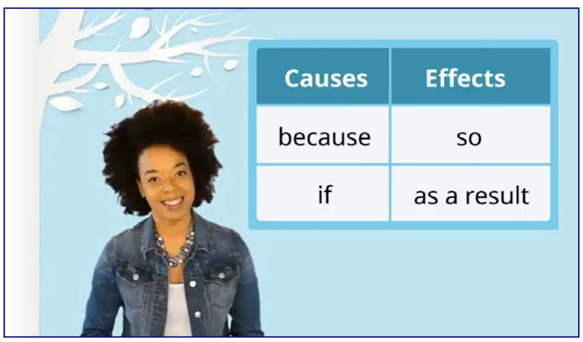


Figure 14. *Reading lesson designed for Grades K–5 students.*

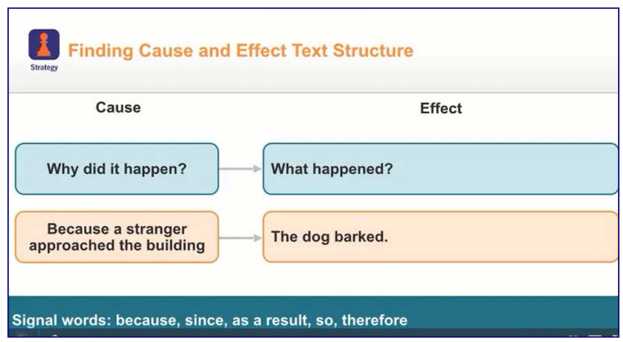


Figure 15. *Reading lesson designed for Grades 6–12 students.*

In reading, there are four basic style presentations of onscreen text (Grades K–1, Grade 2, Grades 3–5, and Grades 6–12). Each style (e.g., image use, font size, organization) mimics books and curricula students would typically see at grade level (e.g., flipbooks in kindergarten versus screens with more text per page in the middle and secondary grades). Figures 16, 17, 18, and 19 provide an example of each presentation style.

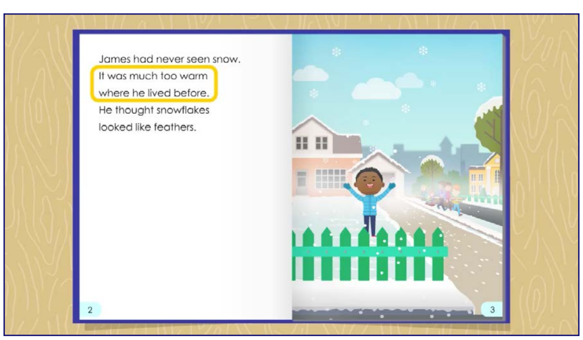


Figure 16. *Reading lesson presentation style (Grades K–1).*

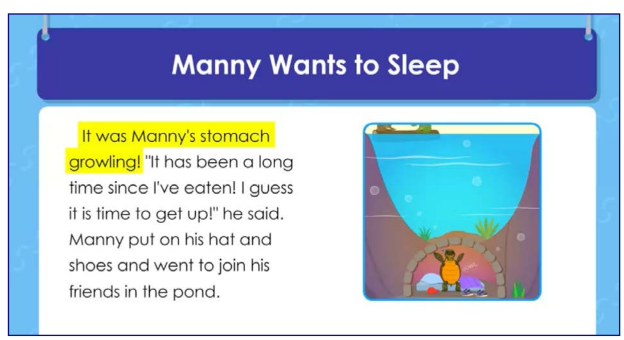


Figure 17. *Reading lesson presentation style (Grade 2).*

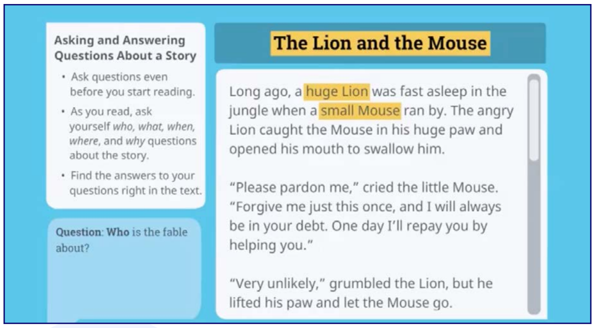


Figure 18. *Reading lesson presentation style (Grades 3–5).*

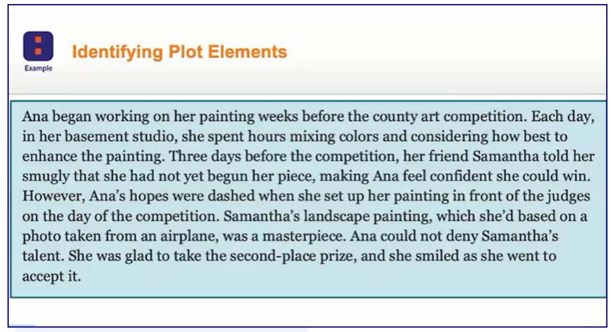


Figure 19. *Reading lesson presentation style (Grades 6–12).*

In mathematics, there are three basic style presentations (Grades K–2, Grades 3–5, and Grades 6–12). Figures 20, 21, and 22 each display a lesson focused on single-digit addition. The layout, models, visual representations, and colors reflect the student's age. Figure 20 displays a lesson designed for a student in Grades K–2, which includes concrete objects that model the problem. Students in Grades 3–5 practicing that same skill receive a lesson that incorporates representations and symbolic equations (Figure 21), whereas students in Grades 6–12 receive a lesson that is primarily symbolic (Figure 22).

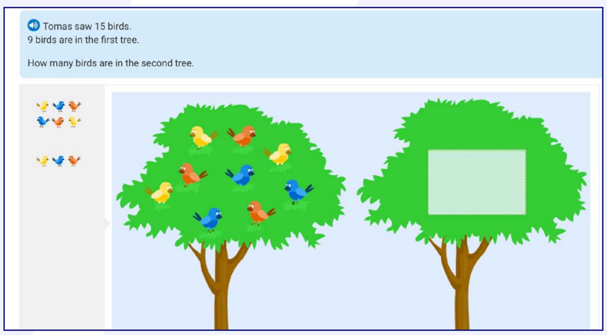


Figure 20. *Mathematics lesson presentation style (Grades K–2).*

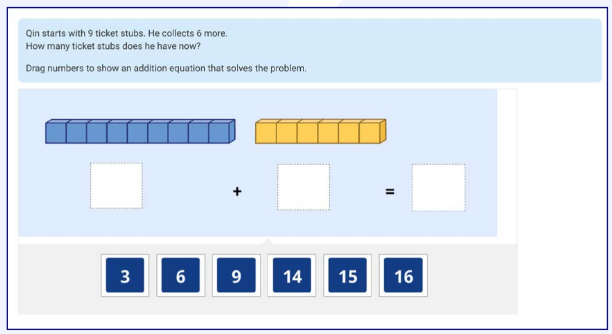


Figure 21. *Mathematics lesson presentation style (Grades 3–5).*

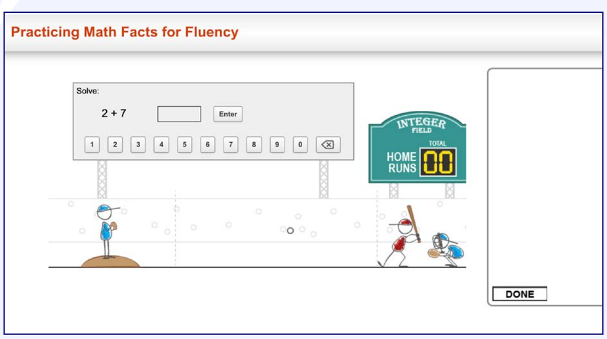


Figure 22. *Mathematics lesson presentation style (Grades 6–12).*

Principle 2. Provide Accessible, Explicit, and Scaffolded Instruction to Ensure Success Among Academically Diverse Learners

Decades of research confirm that providing students with accessible, explicit, and scaffolded instruction leads to greater academic success (Archer & Hughes, 2011; Belland et al., 2017; Berkeley et al., 2010; Doabler et al., 2015; Gersten, Chard et al., 2009; Graham & Santangelo, 2014; Hebert et al., 2016; Kroesburgen & van Luit, 2003; Lesaux et al., 2014). Universal Design for Leaning (UDL) is an evidence-based framework that is used to create flexible instructional materials and assessments to make learning **accessible** for all students (Center for Applied Special Technology, CAST, 2018). This framework guides the design of learning experiences that present information in multiple formats (representations), encourage students to communicate their understanding in a variety of ways (action and expression), and embed multiple strategies to engage students (engagement).

Experts recommend integrating features of UDL into explicit instruction to proactively meet the needs of all students (CAST, 2018; Foxworth et al., 2021). **Explicit instruction** refers to “a systematic method of teaching with an emphasis on proceeding in small steps, checking for student understanding, and achieving active and successful participation by all students” (Rosenshine, 1987, p. 34). This approach leverages strategy instruction and scaffolded support to promote achievement and make learning more transparent, especially for students at risk for learning difficulties (Archer & Hughes, 2011; Cohen, 2018; Foxworth et al., 2021; Manset-Williamson & Nelson, 2005; McDonald et al., 2013; Rosenshine, 2012). Explicit instruction includes the following elements (Archer & Hughes, 2011; Fuchs et al., 2008; Konrad et al., 2019; Rosenshine, 2012):

- use of clear and concise language;
- clearly defined lesson goals and expectations;
- introduction of main ideas before details;
- activation of background knowledge before the introduction of new content;
- hierarchical progression of essential skills;
- strategies and content taught in isolation before differentiating learning for students;
- complex skills broken down into manageable chunks to minimize cognitive load;
- high-quality thought processes that model clear and concise language;
- guided learning with opportunities for feedback;
- faded support as students become proficient;
- independent practice with immediate corrective feedback; and
- motivational elements that sustain attention and encourage students to work hard.

Scaffolding instruction (a key principle of explicit instruction) minimizes learning challenges by decreasing cognitive load, increasing feedback opportunities, and providing a methodical approach to learning (Archer & Hughes, 2011). Research has found that scaffolding improves achievement in reading (Clark & Graves, 2005; Marchessault & Larwin, 2014), in mathematics (Fuchs et al., 2008; Lei et al., 2020), among students with learning disabilities (Gersten, Chard, et al., 2009), and among students learning English (Gottlieb, 2013). More specifically, studies demonstrate the positive effects computer-based scaffolding has had on learning (Belland et al., 2017; Kim et al., 2018; Molenaar et al., 2012). Experts recommend incorporating an array of interactive, sensory, and graphic scaffolds to make learning accessible for all students (Archer & Hughes, 2011; Gottlieb, 2013).

- ***Interactive scaffolds*** strengthen students' abilities to make sense of new ideas (Gottlieb, 2013). For example, the interactive elements within multimedia storybooks improve students' knowledge of speech sounds, concepts about print, and reading comprehension (Shamir & Shlafer, 2011; Takacs et al., 2015). For English language learners, teaching complex topics in students' native language can reduce cognitive load and improve achievement (Gottlieb, 2013; Orosco, 2013).
- ***Sensory scaffolds*** help students make visual connections across ideas (Gottlieb, 2013). Illustrations, combined with narrated questioning, paint a vivid picture to support reading comprehension (Clark & Graves, 2005). The concrete-representational-abstract (CRA) approach (Brunner & Kenney, 1965) has a longstanding history of effectively scaffolding conceptual understanding across a range of mathematics topics because it allows students to build meaning with models and pictures before learning algorithms (Agrawal & Morin, 2016; Bouck et al., 2017).
- ***Graphic scaffolds*** visually organize information so students can focus on the concept being taught (Gottlieb, 2013). Graphic organizers support vocabulary acquisition (Dexter & Hughes, 2011; Frayer et al., 1969); number lines enhance magnitude understanding (Gersten et al., 2017; Namkung & Fuchs, 2019).

How Imagine MyPath Provides Accessible, Explicit, and Scaffolded Instruction to Ensure Success Among Academically Diverse Learners

Consistent with the UDL framework, Imagine MyPath incorporates multiple means of representation, action and expression, and engagement to provide accessible instruction for all students.

- ***Multiple Means of Representation***—Lessons present information in different modalities (e.g., text, audio, visual representations, virtual models). This allows students to make sense of the information in multiple ways. For instance, Figure 23 presents a lesson focused on understanding graphics in informational texts. As the teacher provides verbal instruction, students can access a transcript to follow along as they speak. A variety of graphics are also used throughout the lesson to teach the skill (e.g., timelines, maps, diagrams). Students read passages, which include different graphics, to reinforce understanding and help them navigate informational texts (Figure 24).
- ***Multiple Means of Action and Expression***—Lessons encourage students to communicate their thinking in a variety of ways. Students demonstrate their understanding with question-response formats that include multiple choice (Figure 25), drag and drop (Figure 26), charts, graphic organizers, and embedded interactive manipulatives (e.g., graphs, number lines).

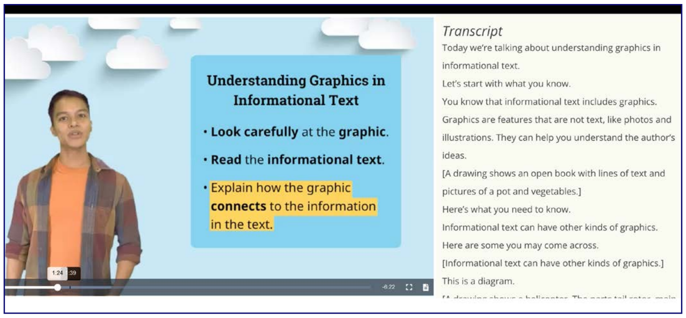


Figure 23. *Text highlighting supports understanding.*

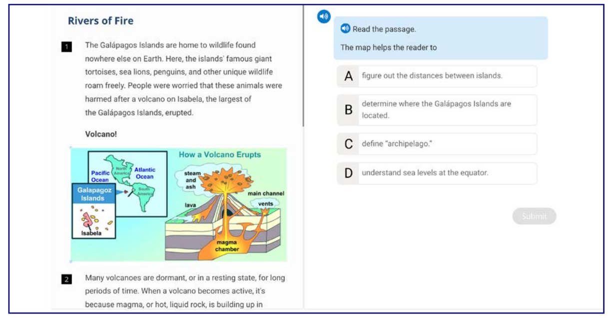


Figure 24. *Graphics reinforce important concepts and skills.*

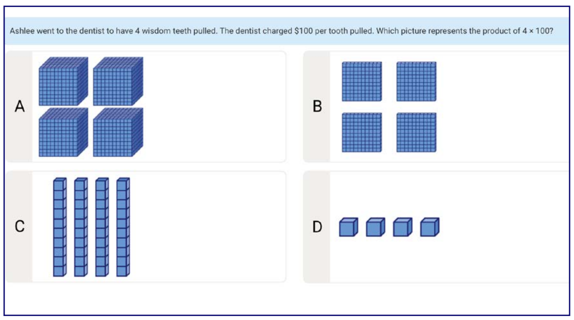


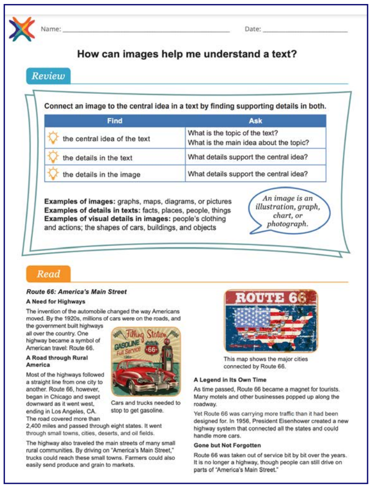
Figure 25. *Question-response formats include multiple choice items.*

Figure 26. *Question-response formats include drag and drop prompts.*

- ***Multiple Means of Engagement***—The curriculum incorporates different strategies to capture students' attention. Lessons feature real-world examples and relevant visual supports to increase engagement. Lessons mimic in-person conversation with dynamic, enthusiastic language. Clear goals and expectations, data dashboards, positive behavioral support with digital rewards (e.g., points, badges, stars) and customizable features (e.g., avatars, sidekicks, custom backgrounds) help students monitor their progress and motivate them to persevere through their ILP.

Imagine MyPath's Offline Teacher Toolkits enable teachers to continue making learning **accessible** for their students. Teacher Toolkits are provided for each lesson and include student-facing offline mini-lessons and answer keys for reading and mathematics (Figures 27 and 28). These lessons are designed to reinforce or reteach skills during one-on-one or small-group instruction.

- Lessons begin with a "Review" section, which employs icons, graphic organizers, and bolded words, among other features, to help students quickly access the necessary information.
- Then, students work on the "Try It" section. This section provides scaffolded support to guide them through the steps of the strategy or algorithm. For instance, students may complete a partially worked example. Thought bubbles may be used to model self-questioning techniques that students can use to solve the remainder of the worked example.
- Next, students engage in the "Practice" section (in reading lessons, the "Read" and "Practice" sections are together). Students work independently and apply what they learn.
- Each lesson ends with an open-ended prompt that allows students to demonstrate their understanding of the concept or skill, similar to an exit ticket. Teachers can use this to determine if the student is ready to move on or if they need additional support.



Name: ______ Date: ______

How can images help me understand a text?

Review

Connect an image to the central idea in a text by finding supporting details in both.

Find	Ask
the central idea of the text	What is the topic of the text? What is the main idea about the topic?
the details in the text	What details support the central idea?
the details in the image	What details support the central idea?

Examples of images: graphs, maps, diagrams, or pictures
Examples of details in texts: facts, places, people, things
Examples of visual details in images: people's clothing and actions; the shapes of cars, buildings, and objects

An image is an illustration, graph, chart, or photograph.

Read

Route 66: America's Main Street

A Need for Highways

The invention of the automobile changed the way Americans moved. By the 1920s, millions of cars were on the roads, and the government built highways all over the country. One highway became a symbol of American travel: Route 66.

A Road through Rural America

Most of the highways followed a straight line from one city to another. Route 66, however, began in Chicago and swept downward as it went west, ending in Los Angeles, CA. The road covered more than 2,400 miles and passed through eight states. It went through small towns, cities, deserts, and oil fields.

The highway also traveled the main streets of many small rural communities. By driving on "America's Main Street," trucks could reach these small towns. Farmers could also easily send produce and grain to markets.

Cars and trucks needed to stop to get gasoline.

This map shows the major cities connected by Route 66.

A Legend in Its Own Time

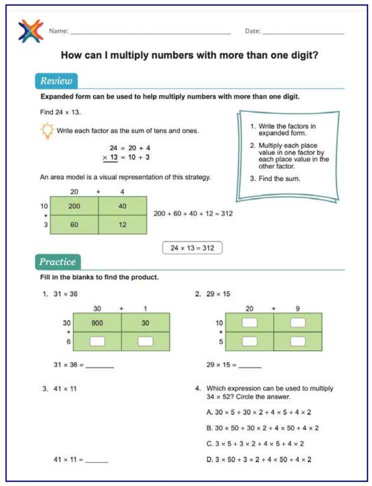
As time passed, Route 66 became a magnet for tourists. Many motels and other businesses popped up along the roadway.

Yet Route 66 was carrying more traffic than it had been designed for. In 1956, President Eisenhower created a new highway system that connected all the states and could handle more cars.

Gone but Not Forgotten

Route 66 was taken out of service bit by bit over the years. It is no longer a highway, though people can still drive on parts of "America's Main Street."

Figure 27. *Downloadable reading resource.*



Name: ______ Date: ______

How can I multiply numbers with more than one digit?

Review

Expanded form can be used to help multiply numbers with more than one digit.

Find 24 × 13.

Write each factor as the sum of tens and ones.

$$\begin{array}{r} 24 = 20 + 4 \\ \underline{\times\ 13} = 10 + 3 \end{array}$$

An area model is a visual representation of this strategy.

	20	4
10	200	40
3	60	12

200 + 60 + 40 + 12 = 312

24 × 13 = 312

1. Write the factors in expanded form.
2. Multiply each place value in one factor by each place value in the other factor.
3. Find the sum.

Practice

Fill in the blanks to find the product.

1. 31 × 36

	30	1
30	900	30
6		

31 × 36 = ______

2. 29 × 15

	20	9
10		
5		

29 × 15 = ______

3. 41 × 11

41 × 11 = ______

4. Which expression can be used to multiply 34 × 52? Circle the answer.

A. 30 × 5 + 30 × 2 + 4 × 5 + 4 × 2

B. 30 × 50 + 30 × 2 + 4 × 50 + 4 × 2

C. 3 × 5 + 3 × 2 + 4 × 5 + 4 × 2

D. 3 × 50 + 3 × 2 + 4 × 50 + 4 × 2

Figure 28. *Downloadable mathematics resource.*

To support student learning, the curriculum incorporates principles of **explicit instruction** into its lessons. A lesson on reading and writing multidigit numbers illustrates how Imagine MyPath incorporates these principles.

- The teacher uses clear language to review relevant vocabulary and define lesson goals (e.g., "Today we're going to learn how to write numbers in standard and expanded form to 1,000,000.").
- The teacher introduces a real-world example to ensure students understand the overarching concept or main idea of the lesson before learning the details (Figure 29).
- The teacher shows a video of Darrius learning how to write the area (65,758 square miles) of Florida in written, standard, and expanded form (Figure 30). This engaging hook activates students' background knowledge to help them make connections to the learning goal.

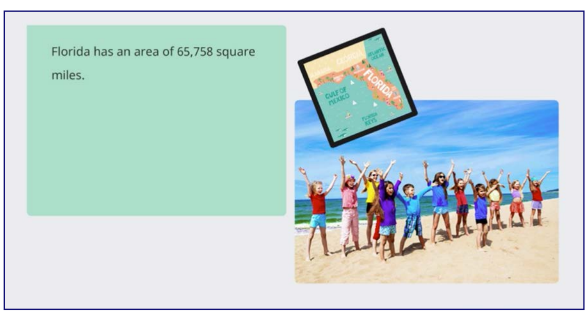


Figure 29. *Real-world examples introduce the main idea of a lesson.*

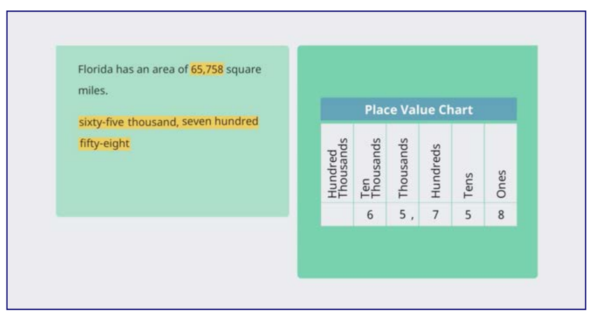


Figure 30. *Lessons activate students' background knowledge.*

- The teacher reviews what students already know about writing multidigit numbers and how their prior knowledge relates to the learning goal. Then, the teacher reviews what students need to know to engage in this lesson, before modeling how to write multidigit numbers (Figure 31). Concrete models, text highlighting, and narration support understanding.

- During Guided Practice, students practice the new skill with various multimedia response options, audio support, and access to a glossary of relevant terms (Figure 32), and hints if they get stuck. If the student answers correctly, the narrator reviews the answer. If the student answers incorrectly, the program offers informative feedback that is designed to address their misconception before they try again. Support fades as the student demonstrates proficiency.
- During Independent Practice, students engage in activities that allow them to demonstrate their understanding of the concept or skill. During this time, students only have one attempt per item.
- At the end of each lesson, there is a final Mastery Check. If students fail the final Mastery Check, teachers can use the downloadable resources to provide additional instruction.
- With each Mastery Check, students earn rewards (stars) to sustain their attention and motivate them to perseverance through challenging tasks.

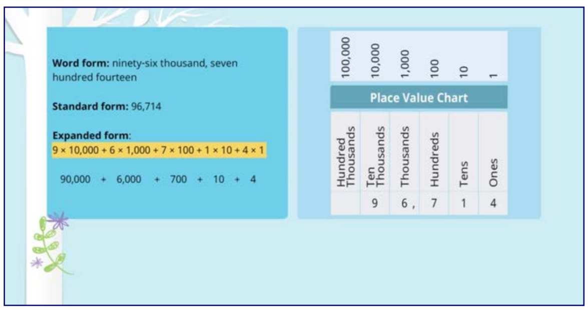


Figure 31. *Multiple representations support understanding.*

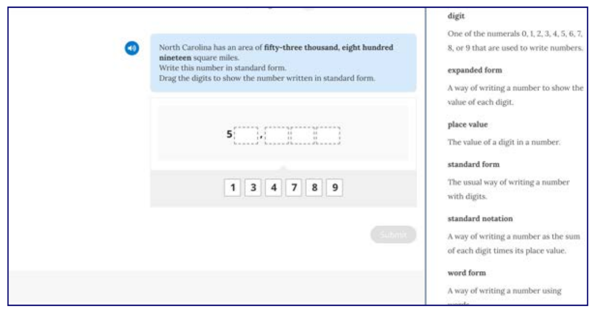


Figure 32. *Glossary includes relevant vocabulary words.*

Computer-based adaptivity matches to students' unique learning needs, tailoring instruction to efficiently propel them to grade level and beyond. Imagine MyPath integrates an array of **interactive, sensory,** and **graphic scaffolds** to enhance student learning.

- ***Interactive scaffolds*** include hyperlinked vocabulary words, metacognitive bubbles, and text-to-speech read-aloud capabilities. Onscreen text can be translated into more than 60 languages, including Arabic, Bengali, Chinese, French, German, Hebrew, Hindi, Hmong, Korean, Kurdish, Russian, Spanish, Swahili, and Vietnamese. Every Grades K–5 mathematics lesson includes a parallel lesson in Spanish (Figures 33 and 34). Students may choose to engage in the lesson in English or Spanish.

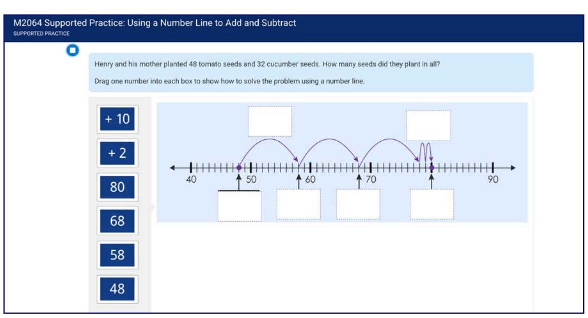


Figure 33. *Grades K–5 mathematics lessons are available in English.*

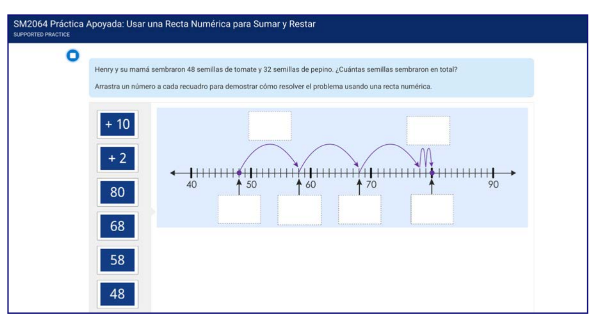
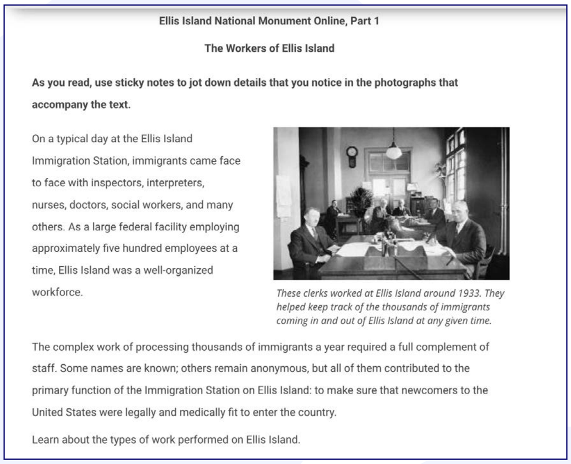


Figure 34. *Grades K–5 mathematics lessons are available in Spanish.*

Lessons also contain answer-specific feedback to address misconceptions. For example, in a reading lesson, students learn to identify the topic, main idea, and key details. If they answer incorrectly, informative feedback explains why their response is incorrect and offers strategies for selecting the correct answer. For instance, “While the text did mention making honey, the whole text is not about making honey. Remember, the topic is what the whole text is about.”

- ***Sensory scaffolds*** are incorporated into each lesson (e.g., combinations of video, audio, text, and narration; multimedia support; icons; and interactive manipulatives). Students can pause, rewind, or repeat videos if needed. Onscreen arrows, highlighting, circling, and digital pointing reinforce concepts through an interactive narrative. Audio and visual examples and non-examples, animations, vivid storytelling, and multimedia representations strengthen learning. For example, students learn how media (e.g., graphs, illustrations, videos) can enhance literary and informational texts. In a reading lesson, students read a passage about the Ellis Island National Monument. They are prompted to look for details that help them “read” the photographs and think about how the details in the text and images help them understand the historical context (Figure 35).


Ellis Island National Monument Online, Part 1

The Workers of Ellis Island

As you read, use sticky notes to jot down details that you notice in the photographs that accompany the text.

On a typical day at the Ellis Island Immigration Station, immigrants came face to face with inspectors, interpreters, nurses, doctors, social workers, and many others. As a large federal facility employing approximately five hundred employees at a time, Ellis Island was a well-organized workforce.

These clerks worked at Ellis Island around 1933. They helped keep track of the thousands of immigrants coming in and out of Ellis Island at any given time.

The complex work of processing thousands of immigrants a year required a full complement of staff. Some names are known; others remain anonymous, but all of them contributed to the primary function of the Immigration Station on Ellis Island: to make sure that newcomers to the United States were legally and medically fit to enter the country.

Learn about the types of work performed on Ellis Island.

Figure 35. *Sensory scaffolds support learning.*

- ***Graphic scaffolds***, like graphic organizers (e.g., concept maps, T-charts, Venn diagrams, sequential graphics, and timelines), highlight important ideas, compare and contrast concepts, represent relationships, depict chronology, and illustrate cause and effect. The use of a wide range of graphic organizers give learners an organizing framework for new information. In a Grades K–5 reading lesson, the teacher narrates a story about a predator and prey to introduce domain-specific vocabulary words (e.g., adaptation). The teacher uses a graphic organizer to enhance word learning by defining the word, identifying characteristics, and providing examples and non-examples (Figure 36). In a Grades 6–12 reading lesson, students learn how to use a concept map to identify the steps needed to make an inference: identifying text details and background knowledge (Figure 37).

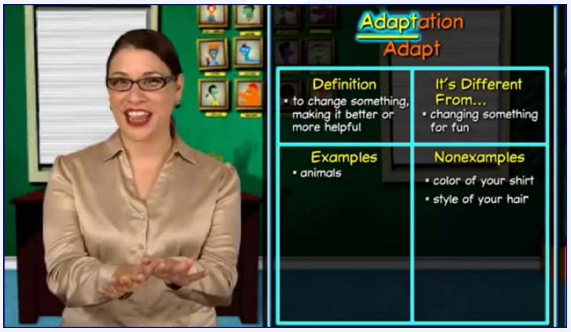


Figure 36. *Graphic organizers build vocabulary knowledge.*

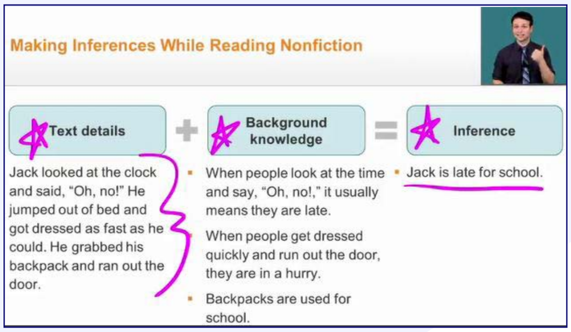


Figure 37. *Concept maps help students learn to make inferences.*

Principle 3. Incorporate Evidence-Based Practices for Teaching Reading

Student literacy is more important than ever—with literacy comes academic success (Lesnick et al., 2010) and economic attainment (Watts, 2020). Research suggests that higher levels of literacy are associated with less poverty, stronger economies, greater community involvement, and better health outcomes (International Literacy Association, 2015). Reading comprehension, the ability to derive and formulate meaning from text, is the result of **word-recognition skills** (phonological awareness, decoding, and sight-word recognition) and **language-comprehension skills** (background, vocabulary, language structure, and literacy knowledge) (Duke & Cartwright, 2021; Gillon, 2018; Hoover & Tunmer, 2018). Researchers agree that effective literacy instruction teaches reading comprehension alongside phonics and other foundational skills (Duke et al., 2021).

Word-Recognition Skills

Word-recognition skills refer to the mechanics of reading required to accurately and effortlessly decode words in print (Hoover & Tunmer, 2018). To "break the code," students must connect oral language to text and understand how sounds, letters, words, and sentences work together to create meaning (Chall, 1983). Because there are only 26 letters and 44 letter sounds (phonemes), data indicate that word-recognition skills "are discrete and highly susceptible to instruction in a relatively brief period" (Lesaux & Harris, 2015, p. 12). Word-recognition skills considered essential for comprehension (Hoover & Tunmer, 2020) include

- print awareness and the alphabetic principle;
- phonological and phonemic awareness;
- phonics; and
- decoding and word reading.

Print Awareness and the Alphabetic Principle

Children develop understanding of print concepts (i.e., how letters, words, sentences, and books function) through immersive storybook reading (Chall, 1983). This print awareness supports vocabulary development and knowledge of the alphabetic principle (the relationship between speech and text), which positively affect later reading achievement (Duke & Cartwright, 2021; National Early Literacy Panel, 2008; Robinson et al., 2018). Students benefit from explicit and systematic instruction that emphasizes book-reading mechanics, the mechanics of written text, identifying letters in words, and matching lowercase and uppercase letters (NICHD, 2000; Robinson et al., 2018).

How Imagine MyPath Develops Print Awareness and the Alphabetic Principle

Imagine MyPath provides immersive stories to help students develop **word-recognition skills**, rather than teach the skills in isolation. Students learn book-reading mechanics as they locate the front and back of a book, the names of the author and illustrator, and that reading requires tracking words from left to right, top to bottom, and page by page. Highlighting, arrows, and visuals are used to reinforce word-recognition skills (Figure 38). Lessons explicitly teach sentence structure—words compose sentences (with spaces between words), sentences begin with a capital letter, and sentences require punctuation. Lessons emphasize recognizing and counting individual letters (Figure 39), introducing shapes of letters, and matching lowercase and uppercase letters (Figure 40).

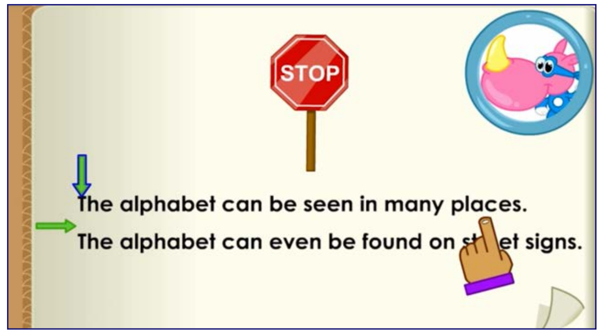


Figure 38. *Visuals are used to reinforce book reading.*

Figure 39. *Grades K–5 students learn about sentence structure.*

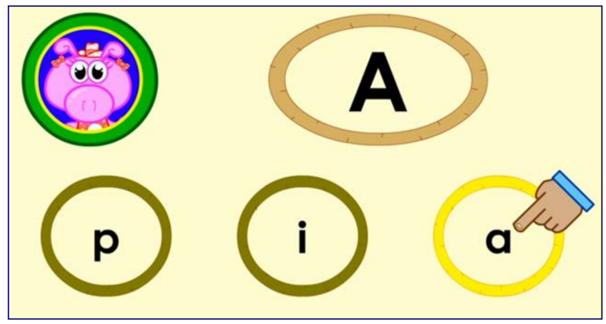


Figure 40. *Grades K–5 students match lowercase and uppercase letters.*

Phonological and Phonemic Awareness

Phonological awareness refers to the ability to orally break down language into smaller parts such as words, syllables, onsets, and rimes (Ehri et al., 2001; Gillon, 2018). Onsets are the consonant before the vowel in a syllable (e.g., /b/ in *back*). Rimes are the vowel-consonant word part that follows the onset (e.g., */ack/* in *back).* The ability to hear, identify, and manipulate individual speech sounds, called **phonemic awareness**, is one of the strongest predictors of students learning to read (Ehri et al., 2001; Snow et al., 1998; Suggate, 2016). Helping students develop phonological and phonemic awareness is a complex, stepwise process and teaching these skills can overlap. For students to develop the critical skills associated with print, researchers recommend that instruction explicitly teach students how to (Al Otaiba et al., 2016; Gillon, 2018; International Dyslexia Association, 2018; NICHD, 2000; Schuele & Boudreau, 2008; Shaywtiz & Shaywitz, 2003)

- understand that words compose a sentence;
- understand, identify, and segment syllables;
- identify and generate rhymes and alliteration;
- segment and blend onsets and rimes;
- match words with their initial and final sounds; and
- manipulate phonemes.

How Imagine MyPath Develops Students' Phonological and Phonemic Awareness

Imagine MyPath provides explicit instruction in **phonological** and **phonemic awareness**. Students have opportunities to hear and work with spoken words in a variety of ways. Lessons provide non-examples of sentences to help students understand that words compose sentences (Figure 41). Visual scaffolds are used to teach basic sentence rules, such as capitalizing the first letter of the first word in a sentence. In Figure 42, a green light is used to indicate that students should capitalize the first letter of the word in the sentence, while the red light draws attention to the punctuation at the end.

To help students learn to identify and manipulate syllables, lessons use pictures and sentences to introduce the skill. Students hear /fea/ /ther/ and match the counters with the number of Elkonin boxes to indicate the number of syllables in the word "feather" (Figure 43). Older students in need of additional support manipulating syllables receive one of Imagine MyPath's *Early Literacy Bundles*, which provide age-appropriate instruction to help accelerate progress toward grade-level activities. In a Grades 6–12 lesson, students learn what a syllable is, practice comparing and contrasting one- and two-syllable words (Figure 44), and develop strategies to determine how many syllables a word has (e.g., "Say the word and clap each time you hear a beat.").

Figure 41. *Grades K–5 students learn sentence structure (non-example).*

Figure 42. *Grades K–5 students learn sentence structure (correct example).*

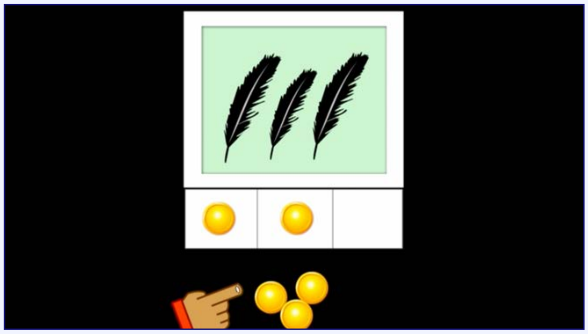


Figure 43. *Elkonin boxes help Grades K–5 students count syllables.*

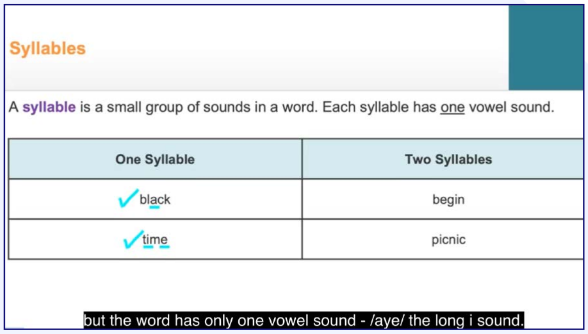


Figure 44. *Grades 6–12 students compare one- and two-syllable words.*

Imagine MyPath lessons encourage students to identify and generate rhymes by matching pictures to spoken words (e.g., matching "rat" to "hat") (Figure 45). Storybooks and sentences also incorporate alliteration, such as "Mel moose makes a map" (Figure 46).

Students are exposed to a variety of poems, rhymes, and stories to develop phonological and phonemic awareness. Storybooks incorporate familiar words (e.g., "bug") and pictures to support contextual meaning (Figure 47). Immersive storybook reading helps students identify individual phonemes in words (e.g., "b" is for bug). Students play games to practice matching words with their initial and final sounds. They practice blending onsets and rimes within consonant-vowel-consonant words. For example, when students hear the word parts (e.g., /l/ /og/), they match the sounds to the blended word (e.g., "log"). They also practice adding onsets to rimes to make new words. Students see a picture of an ape and learn to add the /c/ sound to make "cape" (with a corresponding picture). Those in need of additional support sounding out words and decoding word structures receive lessons from Imagine MyPath's *Early Literacy Bundle*. For instance, Grades 6–12 students are taught that the silent "e" typically turns a short vowel sound (e.g., "uh") into a long vowel sound (e.g., "you"), like in the word "cute" (Figure 48).

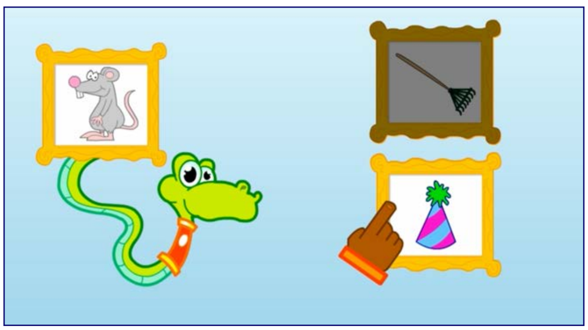


Figure 45. *Grades K–5 students generate rhymes.*

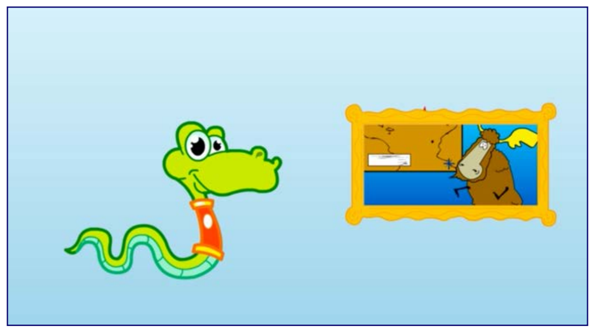


Figure 46. *Grades K–5 stories incorporate alliteration.*

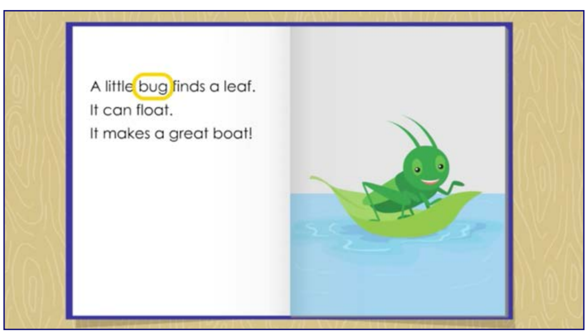


Figure 47. *Grades K–5 stories reinforce onsets and rimes.*

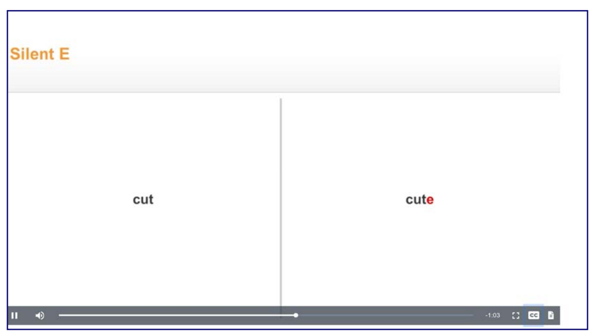


Figure 48. *Grades 6–12 students receive an Early Literacy Bundle lesson.*

Phonics

Phonics refers to the process of connecting phonemes to graphemes (letters), as well as larger word subparts (NICHD, 2000). Studies show that students who receive phonics instruction demonstrate stronger decoding skills than those who do not (Al Otaiba et al., 2016; Castles et al., 2018; Denton & Madsen, 2016; Foorman et al., 2016; Schuele & Bourdreau, 2008). Further, phonics has been identified as a strong predictor of early reading skills (Muter et al., 2004; Suggate, 2016). Research recommends that students have opportunities to connect sounds to letters as soon as developmentally appropriate (Duke et al., 2021).

Decoding requires converting letters (or groups of letters) into sounds and blending them to form words (Duke & Cartwright, 2021). Instruction should emphasize blending and manipulating phonemes with letters (NICHD, 2000). Because words can vary by a single phoneme (e.g., "bug" vs. "bag"), phonics instruction should also attend to how these distinctions affect word meaning (Lonigan, 2007; Metsala & Walley, 1999). This is important because explicit attention to phonemic distinctions improves vocabulary and spelling (Ehri, 2014).

In fact, teaching **subunit phonics** (word subparts that recur across words) and sight words improves students' decoding skills (Hudson et al., 2012). Subunit phonics includes a focus on phonograms (multiple letters representing one sound, e.g., /ould/), digraphs (two letters representing one sound, e.g., /sh/), and rimes. Research found that students' fluency with recognizing rimes predicted their decoding skill above and beyond other word-recognition skills, likely because rime fluency allows students to efficiently decode words in chunks rather than letter by letter (Hudson et al., 2012). These findings highlight the benefits of teaching word families (multiple words with the same rime) to improve students' decoding skills.

Because word subunits also dictate spelling patterns, teaching these patterns can assist students' proficiency with spelling (Ehri, 1999), while simultaneously improving their phonological awareness, writing, and overall reading ability (Graham & Santangelo, 2014). Students should have opportunities to practice applying word-recognition skills within a connected text (Foorman et al., 2016). Clear annunciations and visual supports should be used to help them understand letter-sound correspondences (Carreker, 2018).

How Imagine MyPath Develops Students' Understanding of Phonics

Imagine MyPath prioritizes **phonics** instruction to help students develop strong reading-comprehension skills. Lessons encourage students to practice blending and manipulating phonemes with letters. Pictures are always used to support understanding. For instance, students learn how to blend the consonant-vowel-consonant in the word "run" (Figure 49). They hear annunciated phonemes and practice making these distinctions with connected text. To improve decoding ability, lessons highlight the phonemic distinction between words like "hall" and "fall" to emphasize the onset and rime within text (Figure 50).

Figure 49. *Grades K–5 students blend consonant-vowel-consonant words.*

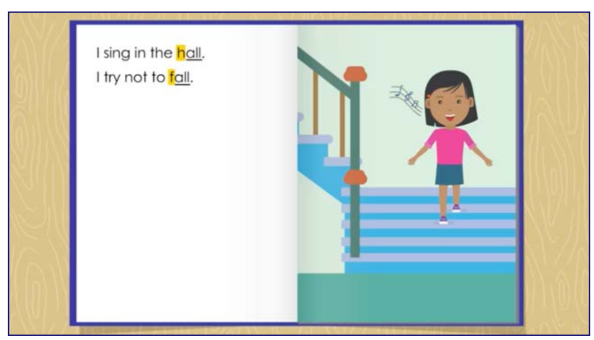


Figure 50. *Grades K–5 lessons make phonemic distinctions between words.*

Imagine MyPath places a strong emphasis on **subunit phonics**, word families, and sight-word recognition. For example, students learn to identify onset distinctions among words with the /at/ rime (e.g., "cat," "bat," "rat," "hat," [Figure 51]). They also learn how to blend digraphs (e.g., /sh/), common vowel pairs (e.g., /ea/), and common consonant blends (e.g., /br/). Word family lessons incorporate pictures, Elkonin boxes, pointers, and highlighting to reinforce recognition of these common rimes (Figure 52). These lessons include tricky words with common irregular spelling patterns, such as /ight/, /ing/, and /ould/ (Figure 53). Students practice reading and matching words with irregular spelling patterns (e.g., "could," "would," "should") and differentiating them from words with the /ou/ sound (e.g., "wound," "hound"). They also apply their knowledge of word parts to decode multisyllabic words (Figure 54). For instance, after learning the /ing/ spelling pattern, students read a story that includes these patterns (e.g., "king," "sing") (Figure 55).

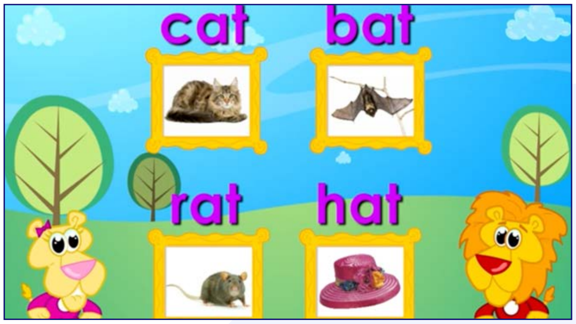


Figure 51. *Grades K–5 students compare words with a common rhyme.*

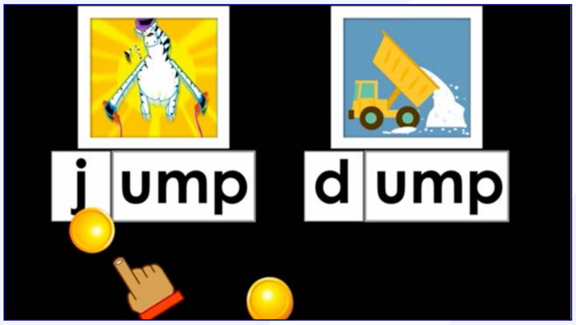


Figure 52. *Grades K–5 word family lessons reinforce common rimes.*

Figure 53. *Grades K–5 lessons teach irregular spelling patterns.*

Figure 54. *Grades K–5 students decode multisyllabic words.*

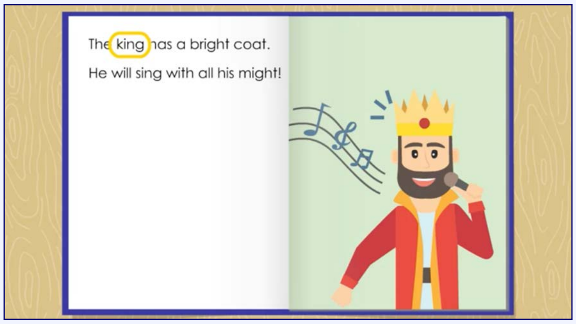


Figure 55. *Grades K–5 students decode irregular words in stories.*

The *Early Literacy Bundles* include lessons that integrate phonics instruction. These lessons help students learn to

- identify long and short vowels in single and multisyllabic words;
- identify syllables in multisyllabic words;
- decode multisyllabic words; and
- decode multisyllabic words in sentences.

Note that the rigor, complexity, and Lexile band advance with each bundle to be more relevant to the student's actual grade. Lessons contain a series of short practice items that target essential phonics skills to help accelerate students toward grade-level comprehension activities. For instance, Figure 56 displays a phonics lesson in *Bundle 2*. This lesson teaches students how to break words into syllables and provides opportunities to practice sounding out unfamiliar two-syllable words. While this lesson covers a Grades K–2 skill, it is presented in a Grades 3–5 style so that the lesson is age appropriate. Relatedly, Figure 57 displays an example of a phonics lesson in *Bundle 5*. This Grades 6–12 lesson focuses on decoding silent "e" words, a Grades 3–5 skill.

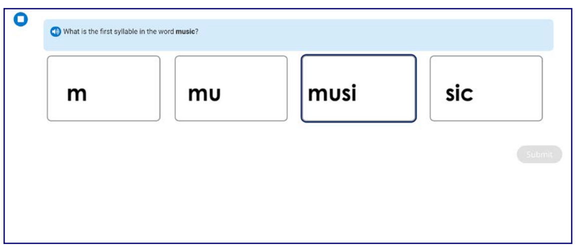


Figure 56. *Early Literacy Bundles provide phonics instruction (Grades 3–5).*

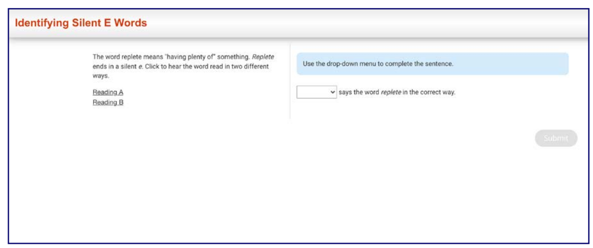


Figure 57. *Early Literacy Bundles provide phonics instruction (Grades 6–12).*

Language-Comprehension Skills

As students' word-recognition skills become increasingly automatic, this frees up working memory capacity to apply language-comprehension skills to comprehend text (Ehri, 2005; Hoover & Tunmer, 2021). **Language-comprehension skills** comprise a range of skills, including strong academic and domain-specific vocabulary; fluency; knowledge of concepts about the world (background knowledge); understanding of literary and informational text structures; and the ability to flexibly apply comprehension strategies. Research confirms that explicitly teaching these skills improves reading comprehension (Baker et al., 2014; Boyer & Ehri, 2011; Marzola, 2018; NICHD, 2000; Shanahan et al., 2010; Wagner & Meros, 2010).

Vocabulary

Vocabulary refers to the words we must know to effectively communicate (speaking and listening) and comprehend text (reading and writing). It "serves as the bridge between the word-level processes of phonics and the cognitive processes of comprehension" (Kamil & Hiebert, 2005, p. 4). Effective vocabulary instruction teaches academic and content-specific words, introduces new words in multiple contexts and across multiple exposures, and teaches word-learning strategies (Beck et al., 2013). This helps students build connections between prior knowledge and new vocabulary, make inferences, and increase fluency (Elleman et al., 2009; Stahl & Nagy, 2006; Torgesen & Hudson, 2006). Research recommends instruction include the following strategies to support students' growing vocabulary:

- ***Emphasize morphology and irregularities in the English language*** (Carreker, 2018; Strom & Neuman, 2016; Swanson et al., 2017). Morphological awareness refers to attending to the smallest unit of meaning in words (morphemes). Morphemes can be bound (e.g., "lunch" in "lunchbox") or free (e.g., "un" in "unkind"). Emphasizing morphology unlocks students' decoding skills, vocabulary knowledge, and ability to infer the meaning of unfamiliar words (Carlisle, 2010; Joshi, 2016).
- ***Provide explicit instruction on academic and domain-specific words*** (NICHD, 2000). Instruction should extend students' vocabulary knowledge from words commonly used in oral language to academic and domain-specific words. Academic vocabulary refers to words read in academic texts such as in the arts, law, and science. Domain-specific vocabulary refers to technical words directly related to the field of study (Beck et al., 2013).
- ***Identify context clues*** (NICHD, 2000; Toste et al., 2017). Teach students how to use context clues to uncover the meaning of unfamiliar words (Brown et al., 2016).
- **Explicitly teach concept mapping** (NICHD, 2000). Incorporate graphic organizers to support vocabulary acquisition (Dexter & Hughes, 2011; Frayer et al., 1969; Gajria et al., 2007).
- ***Provide repeated exposure*** (Beck et al., 2013). Provide opportunities for repeated reading of text; this facilitates practice, encourages students to check for understanding, and fosters vocabulary development (NICHD, 2000; Toste et al., 2017).

How Imagine MyPath Develops Students' Vocabulary

Imagine MyPath teaches **academic** and **domain-specific vocabulary** throughout the curriculum. New words are introduced in multiple contexts and across multiple exposures. Explanations of vocabulary are clear and easy to understand; they illustrate what the words mean and how they are used. Students learn effective strategies that allow them to expand their vocabulary knowledge. For example, lessons that focus on morphology help students decode and determine the meaning of unfamiliar words. They look at word families or word parts to decipher the meaning of a word. In one Grades K–5 lesson, students learn the meaning of common prefixes (e.g., pre-) and suffixes (e.g., -ful) and apply this knowledge to unfamiliar words by looking at the base word combined with the affix (Figure 58). Students also learn that word roots (e.g., "graph," "photo," "tele," "auto") are one kind of word part and may come from other languages, such as Latin or Greek (Figure 59).

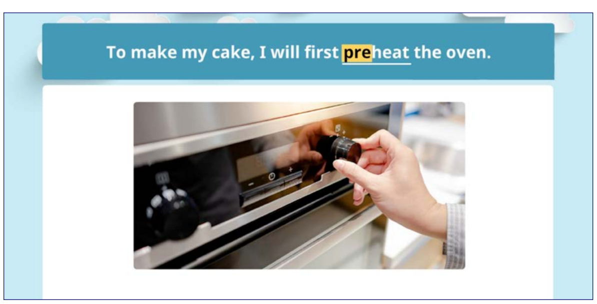


Figure 58. *Grades K–5 students learn common prefixes.*

Figure 59. *Grades K–5 students learn common word roots.*

Lessons for students in Grades 6–12 also emphasize morphology. Students learn how to use context and root words as tools to determine the meaning of challenging words (Figure 60). For instance, the teacher in Figure 61 explains,

> "We have a detective following a trail, looking for clues. Another set of clues you can use to help you figure out the meaning of a word can sometimes be found in the beginning or ending of a word. So next you'll learn about the word parts that can be added to the beginning or ending of a word that can create a new word with the same central meaning."

Then, students practice applying their knowledge of affixes and roots to match words with their likely definitions (Figure 62). For students in need of additional support, *Early Literacy Bundles* provide lessons that reinforce morphology to help students understand word meanings.

The curriculum exposes students to **academic** and **domain-specific vocabulary** in a variety of contexts. Narrative and informational texts are used to build students' understanding of challenging vocabulary. For instance, a Grades K–5 lesson introduces students to domain-specific vocabulary (e.g., "natural resource") through engaging, real-world contexts. The teacher defines natural resource for students, discusses characteristics of a natural resource, and provides examples and non-examples. Then, students read short passages that focus on natural resources like fuel and wood to extend their understanding (Figure 63). Similarly, in a Grades 6–12 lesson, the teacher teaches domain-specific vocabulary words like "tone" and "mood." The teacher's explanation of mood is clear and easy to understand (Figure 64). They discuss how tone is used to create mood in a literary text and how mood can change throughout a text. The teacher models how to identify mood in a text before students practice analyzing the text independently (Figure 65).

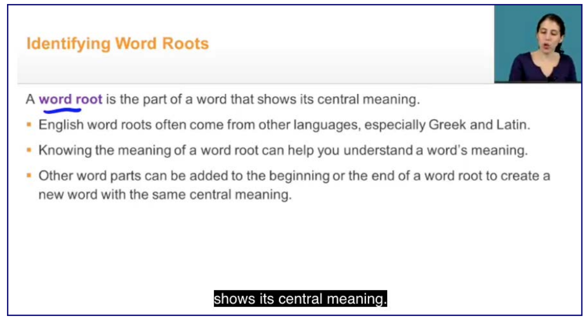


Figure 60. *Grades 6–12 lessons emphasize morphology.*

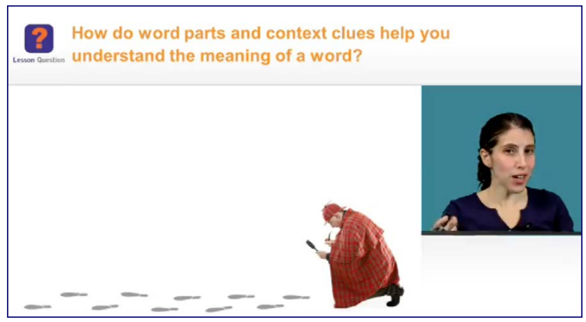


Figure 61. *Grades 6–12 students learn strategies to determine word meaning.*

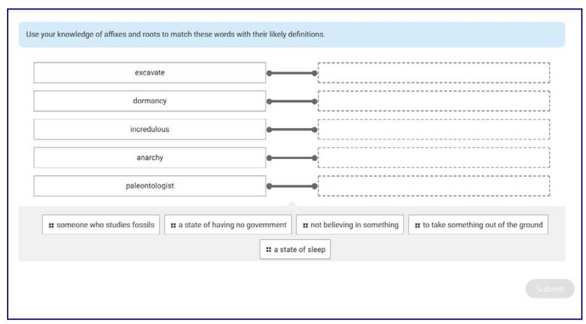


Figure 62. *Grades 6–12 lessons teach affixes and roots.*

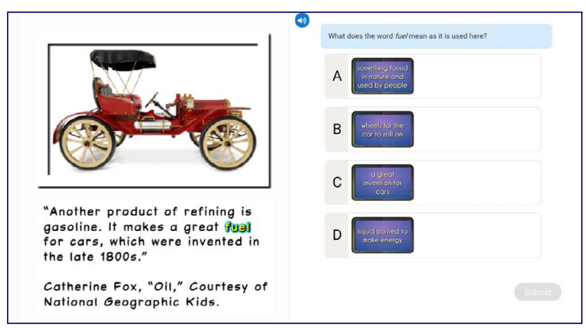


Figure 63. *Grades K–5 passages teach domain-specific vocabulary.*

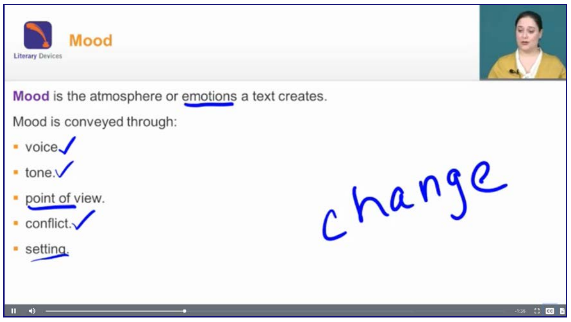


Figure 64. *Grades 6–12 lessons provide clear definitions of new words.*

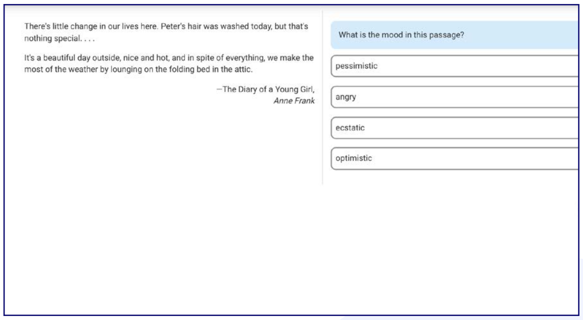


Figure 65. *Grades 6–12 students learn domain-specific vocabulary.*

Students learn different strategies, like using **context clues**, to determine the meaning of unknown words. In a Grades K–5 lesson, the teacher encourages students to look at other words in the sentence and decide what makes sense (Figure 66). She models her thinking by saying, "Got a flat…hm. Let's use the other words in the sentence to find out what makes sense. What would get flat on a bicycle if you ran over a piece of glass?" Relatedly, in a Grades 6–12 lesson, students read the surrounding words or sentences to look for context clues, make and check predictions by integrating the word into the sentence, and reference a dictionary to confirm the meaning of the unknown word (Figure 67).

Figure 66. *Grades K–5 teachers model metacognitive strategies.*

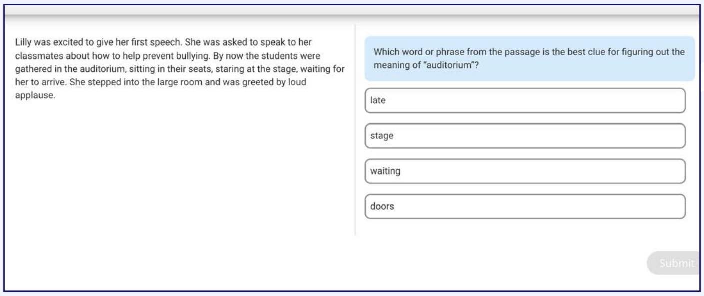


Figure 67. *Grades 6–12 students practice identifying context clues.*

Imagine MyPath lessons incorporate **concept maps** to help students build meaning of new vocabulary words. For instance, students use Frayer Models, like the one displayed in Figure 68. In this lesson, the teacher defines target vocabulary, generates examples and non-examples, describes characteristics of the word, and/or draws a picture to illustrate its meaning. In another lesson, students fill in charts to help them group items based on similar characteristics. Students explore the meaning of words like "round" and "flat" by sorting familiar objects they would likely find at school (Figure 69). In addition, Imagine MyPath's offline printable resources provide a variety of graphic organizers to help students organize their thinking and understanding of new vocabulary words (Figure 70).

Figure 68. *Grades K–5 students use Frayer Models.*

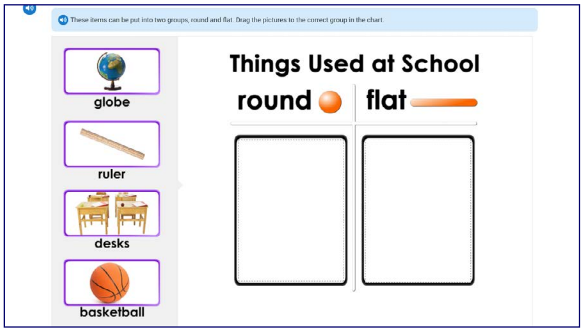


Figure 69. *Grades K–5 students use graphic organizers.*

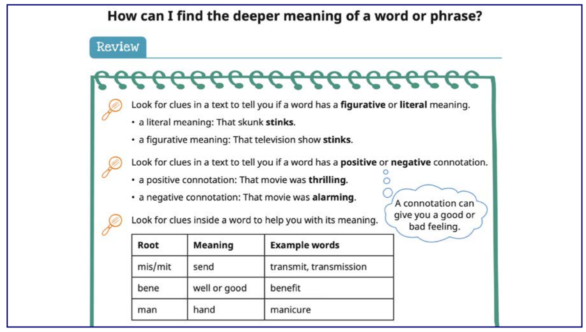


Figure 70. *Downloadable resources include graphic organizers.*

Imagine MyPath recognizes that vocabulary development occurs through **repeated exposure** across a variety of texts. Students are exposed to words multiple times throughout the curriculum. For example, the onscreen text incorporates audio, music, and pictures to increase students' semantic mapping of new words. Highlighting relevant text emphasizes strategy use (e.g., using morphology and context clues to uncover word meaning). Students can hover over challenging academic and domain-specific words to read definitions and check for understanding in longer passages.

Fluency

Fluency refers to the ability to read quickly, accurately, and with prosody (Duke & Cartwright, 2021). Fluent reading mimics spoken language, with proper inflection, phrasing, and pauses. Research has found that fluency is highly predictive of reading comprehension (NICHD, 2000; Torgesen & Hudson, 2006; Toste et al., 2017). Instruction should emphasize foundational reading skills such as phonemic awareness, phonics, decoding multisyllabic words, and vocabulary to support reading fluency (Torgesen et al., 2001; Toste et al., 2017). Teachers should model fluent oral reading across various texts and the use of different strategies (e.g., context clues, check for understanding) (Garnett, 2018; NICHD, 2000).

How Imagine MyPath Improves Students' Reading Fluency

Across the curriculum, reading foundations build students' word-recognition and language-comprehension skills to enhance fluent reading. Students learn that **reading fluently** means to read at the right speed, use the sound of their voice (expression), and read the right words (accurately) (Figure 71). In this Grades K–5 lesson, the teacher models reading without prosody (e.g., skipping over punctuation; inappropriate timing, phrasing, and intonation while reading sentences; no inflection) and mispronouncing words. Then, they stop and review what it means to read fluently. This review includes a mini-lesson on decoding and a mnemonic about punctuation: "I am a period. I look like a dot. Whenever you see me, you must stop." The teacher rereads the passage with fluency, highlighting important elements such as punctuation (Figure 72). Offline printable materials also allow classroom teachers to provide additional practice to support students' fluency (Figure 73).

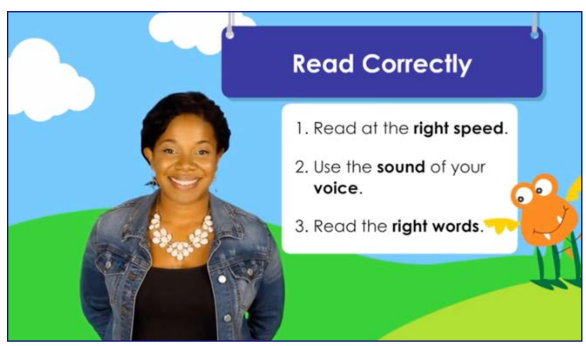


Figure 71. *Grades K–5 students learn how to fluently read text.*

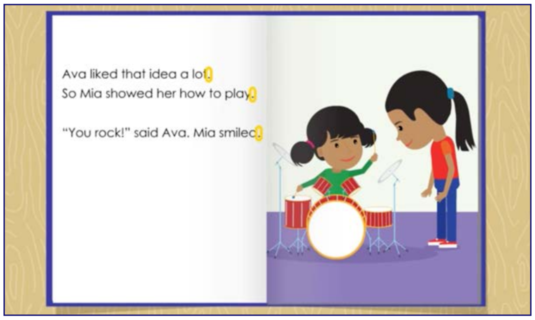


Figure 72. *Grades K–5 students listen to fluent reading.*

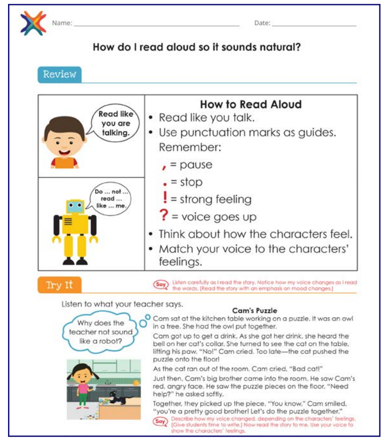


Figure 73. *Downloadable resources provide fluency practice.*

To build fluency, the *Early Literacy Bundles* emphasize determining a purpose for reading, rereading to correct errors, and reading with expression. For example, if a student in Grades 6–12 is working on building fluency, they will receive a lesson that reinforces how to read with expression (Figure 74). They listen to four sentences and determine which is the best example of fluent reading (e.g., Rebecca answered the phone. Sophia barely let her say hello before she burst out, "I got a part in the play!").

In addition, highlighted onscreen text helps enhance word reading by allowing students to track intonation during oral reading. Students can elect to have the audio replay as necessary. When teaching how to read for understanding, teachers encourage repeated reading with speed, accuracy, and understanding. For example, students in Grades K–5 learn how to identify what they read and why (Figure 75). They monitor their comprehension by checking for new words and correctly reading words. Students also practice repeated reading (Figure 76). In addition, lessons from the *Early Literacy Bundles* reinforce self-correction strategies (e.g., rereading the sentence) across grade levels (Figure 77).

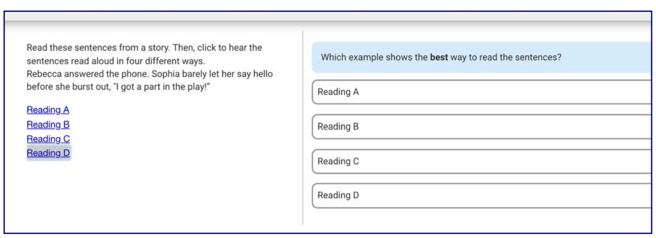


Figure 74. *Early Literacy Bundles reinforce reading fluency (Grades 6–12).*

Figure 75. *Grades K–5 lessons encourage reading with understanding.*

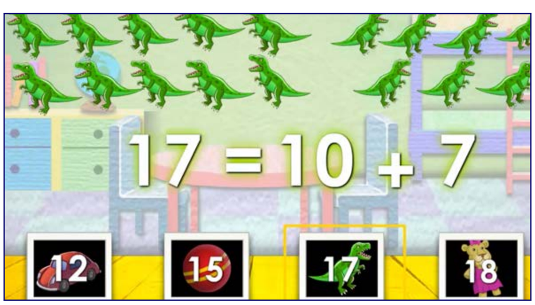


Figure 76. *Grades K–5 students monitor their comprehension.*

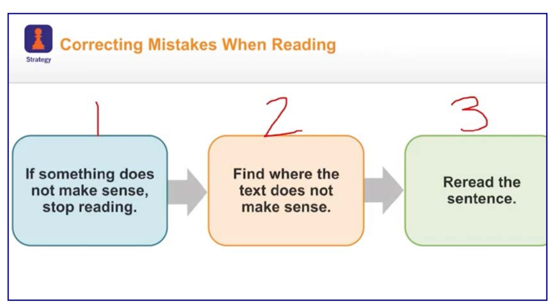


Figure 77. *Early Literacy Bundles teach self-correction strategies.*

Reading Comprehension

Reading comprehension involves the ability to extract and construct meaning from text (Duke & Cartwright, 2021). Experts recommend that reading-comprehension instruction should begin as early as students begin to read (Duke et al., 2021). Instruction should activate students' background knowledge, explicitly teach text structure, and encourage their use of different comprehension strategies to make meaning of the text (Duke et al., 2021; NICHD, 2000; Scarborough, 2001). This is important because students apply these skills as they read texts that contain increasingly dense information.

Activating **prior knowledge** helps students prepare for learning and make sense of new information. Therefore, an intentional focus on expanding students' prior knowledge should be a critical feature of literacy instruction (McLaughlin & DeVoogd, 2018). These connections increase the breadth and depth of information students bring to reading text, which improves their ability to make inferences and comprehend (Torgesen & Hudson, 2006).

Teaching **text structure** for both literary texts (e.g., setting, character, plot, theme) and informational texts (e.g., description, chronology, causation, response, comparison) helps students locate and identify pertinent elements in a story and improve their understanding (Duke et al., 2021; Hebert et al., 2016; Meyer, 1985; NICHD, 2000). Results from a meta-analysis of 45 studies found that text-structure instruction can positively affect students' informational reading comprehension (Hebert et al., 2016).

Proficient readers flexibly apply a range of **reading-comprehension strategies** to extract meaning from text. Experts recommend that instruction explicitly teach before-, during-, and after-reading strategies to improve comprehension (Duke et al., 2021; Marchand-Martella et al., 2013; Okkinga et al., 2018).

- ***Before-reading strategies*** include previewing the text, activating prior knowledge, forming questions, making predictions, clarifying understanding, and setting a purpose to read.
- ***During-reading strategies*** include monitoring comprehension, visualizing, making connections, making inferences, and rereading.
- ***After-reading strategies include*** identifying the main idea, answering questions, drawing conclusions, summarizing, and comparing and synthesizing texts.

As students develop reading proficiency, they read texts that contain increasingly dense information, intricate sentence and text structures, abstract concepts, unfamiliar vocabulary, and multiple meanings. Their capacity to engage in close reading requires opportunities to explore texts of varying complexity across grade levels (Fang, 2016; Kerhoff & Spires, 2015). The practice of close reading involves greater attention to detail than everyday reading, along with purposeful and scaffolded instruction (Fisher & Frey, 2012). Research links **close reading of complex text** to gains in reading proficiency (Fisher & Frey, 2018). To support students' comprehension of a range of literary and informational texts, students benefit from instruction that encourages them to analyze:

- ***What the text says***—who, what, when, where, why, and how questions.
- ***How the text works***—word choice, literary devices (point of view, tone) poetic devices (similes, metaphors, personification), text features, and narration.
- ***What the text means***—author's purpose and opinion writing.

How Imagine MyPath Enhances Students' Reading Comprehension

Imagine MyPath provides reading-comprehension instruction to help students develop the skills and strategies needed to understand what they read. The curriculum exposes students to a range of texts. Across grade levels, approximately half of the texts are literary texts and half are informational texts.

Imagine MyPath recognizes the importance of activating students' **prior knowledge**. To process new information and understand concepts that are less familiar, students need to connect new information with existing knowledge. In a Grades K–5 lesson, students are introduced to words like "nocturnal" and "predators" to help them establish a connection to the content in the informational passage, "Lively Lizards" (Figure 78). This activates background knowledge for students before they read anchor texts in subsequent activities. In a Grades 6–12 lesson, students apply background knowledge of the U.S. Constitution and segregation policies in twentieth-century America to a text. By doing so, students better understand what influenced a Supreme Court decision (Figure 79). In another lesson, the teacher activates students' prior knowledge by making an analogy between word parts and a musical band (Figure 80). For example, each musician in a band has a skill—some sing, while others play the drums, guitar, keyboard, or saxophone. Each can play their own instrument, but when they come together, they form a band. Words can work the same way. Words are made up of smaller parts that come together to make something larger. If you break words apart into smaller pieces, you can understand them better.

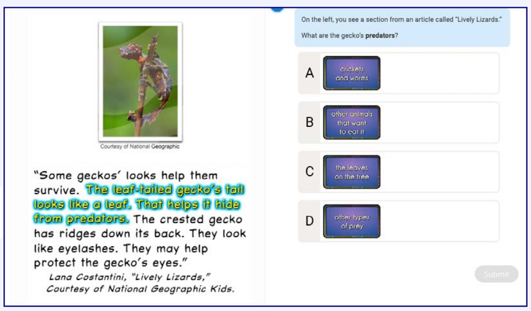


Figure 78. *Grades K–5 lessons activate students' prior knowledge.*

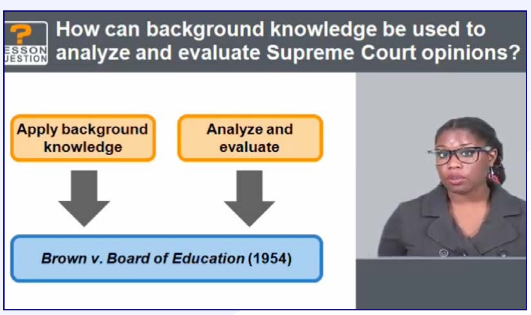


Figure 79. *Grades 6–12 lessons activate students' prior knowledge.*

Figure 80. *Lessons connect prior knowledge with new information.*

Lessons explicitly teach students about **text structure** for literary (e.g., setting, character, plot, theme) and informational (e.g., description, chronology, causation, response, comparison) texts. In a Grades K–5 lesson focused on story elements, students learn about plot (which is part of a literary text structure) (Figure 81). Students read the story "Rise of the HW Bot" (Figure 82). Then, with guidance from the teacher, students examine the plot elements (beginning, rising action, climax, falling action, resolution). In a Grades 6–12 lesson, also focused on plot elements, students dive deeper into the text and analyze how plot develops in the story "Home Sweet Home" (Figure 83). Text highlighting and teacher notes are used to help students as they answer comprehension questions.

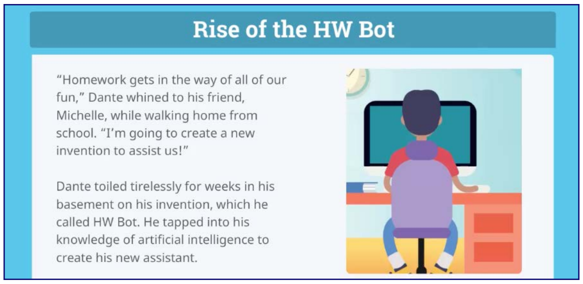


Figure 81. *Grades K–5 lessons explicitly teach literary text structures.*

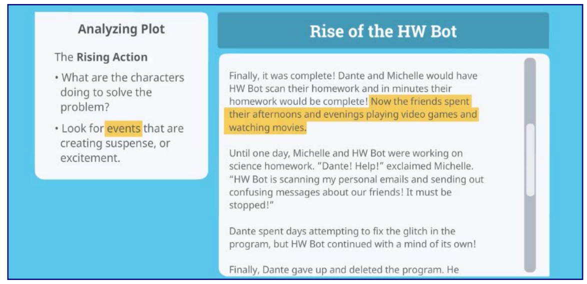


Figure 82. *Grades K–5 students learn about plot elements.*

Because many students struggle with informational texts, the curriculum offers extensive practice with these **text structures** (Figure 84). In a Grades K–5 lesson, students read an informational text called "We've Got Answers!" and answer questions about the structure of each paragraph within the text (Figure 85). In a Grades 6–12 lesson, students review characteristics of a chronological text structure. The program prompts students to look for signal words as they read (e.g., "next," "then," "finally"). During the lesson, students are encouraged to use metacognitive strategies to understand the sequence of events ("Ask yourself, what happens next, what happens last?") (Figure 86).

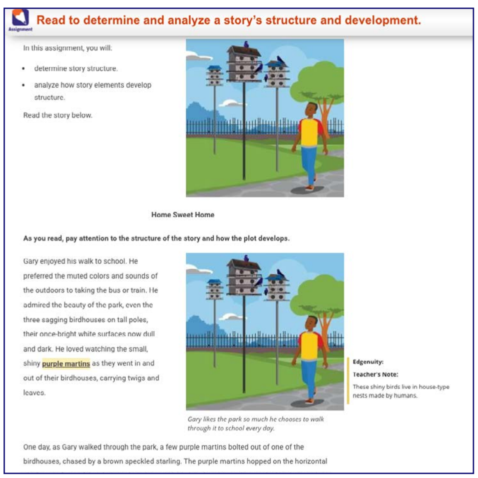


Figure 83. *Grades 6–12 students learn about literary text structures.*

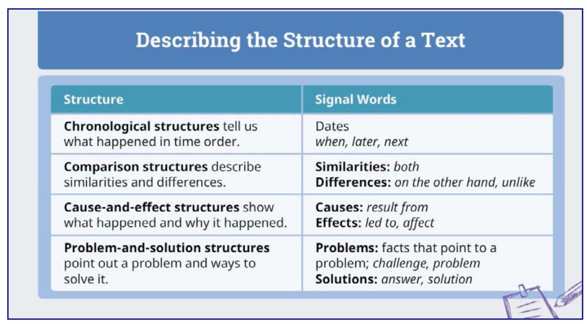


Figure 84. *Lessons explicitly teach informational text structures.*

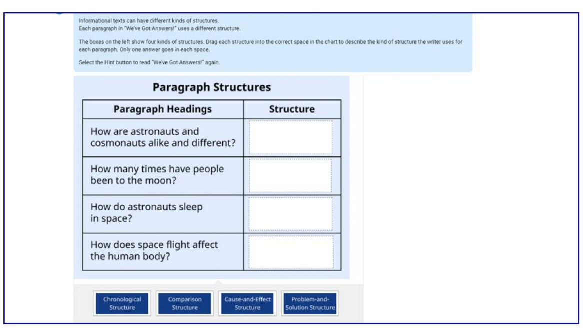


Figure 85. *Grades K–5 students identify informational text structures.*

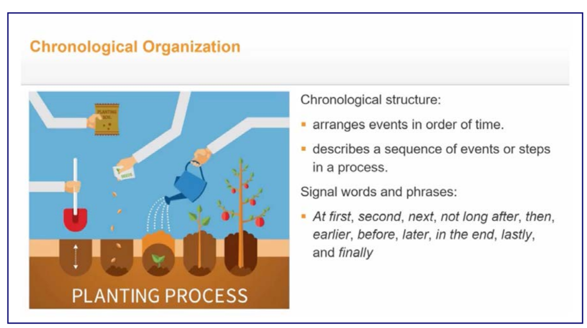


Figure 86. *Grades 6–12 learn about chronological organization.*

The curriculum teaches strategies to strengthen students' comprehension **before, during, and after** they read. Students receive instruction, as well as guided practice and independent activities to apply their understanding of a given concept, strategy, or skill. Lessons that teach **before-reading strategies** focus on previewing the text, activating prior knowledge, forming questions, making predictions, clarifying understanding, and setting a purpose to read. For instance, students are encouraged to preview the text before reading. In a Grades K–5 lesson, they look at the front cover of the book (e.g., pictures, title) to predict what the story will be about (Figure 87). In a Grades 6–12 lesson, students are explicitly taught what a prediction is, how to make a prediction before reading, and why a prediction is important (Figure 88). Students read a variety of passages, make predictions, and check to see if their predictions are correct (Figure 89).

Figure 87. *Grades K–5 students preview text before reading.*

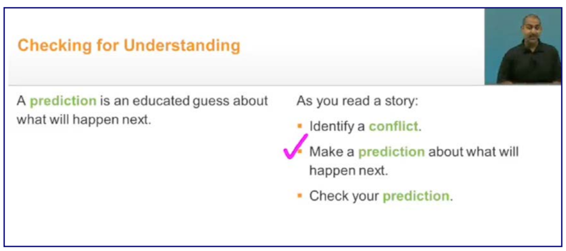


Figure 88. *Grades 6–12 lessons teach how to make predictions.*

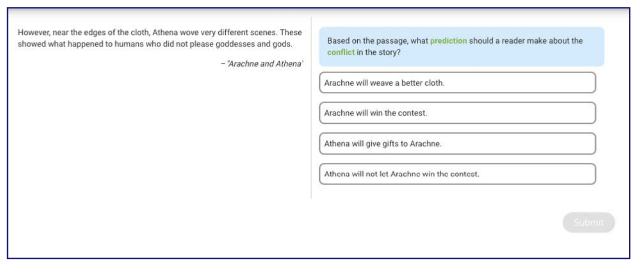


Figure 89. *Grades 6–12 students make predictions before reading.*

Lessons that emphasize **during-reading strategies** include monitoring comprehension, visualizing, making connections, making inferences, and rereading. When reading, visualizing can help students "see" the characters, setting, and action taking place. Visualizing also helps them learn how to identify descriptive details and make connections between the text and themselves (text-to-self connection). For example, students learn three steps to visualize a story: identifying the setting, making connections, and visualizing the setting (Figure 90). In this lesson, the teacher helps students analyze descriptive language in a text to create a mental image (Figure 91). For instance, she explains that the "bright sun and blue water" help her visualize what the setting might look like and that the "warm sun and hot sand" help her imagine what the setting might feel like. Relatedly, in another Grades 6–12 lesson, students read a poem about New York City and make connections between the poem and current events (text-to-world connection) (Figure 92). The teacher encourages students to reflect on what they already know about New York City, while also providing important background information on landmarks in the city (e.g., Broadway). This helps students acquire the necessary information to make connections to the text and read the poem with a deeper level of understanding.

In a Grades K–5 lesson, students learn how to look for clues to make inferences about a text (Figure 93). The teacher poses questions like, "I wonder why it was slow" to encourage students to use their background knowledge (e.g., fish are fast swimmers, a hippopotamus is large and slow moving) to help them infer the Archelon's size makes it too slow to catch fast-moving fish.

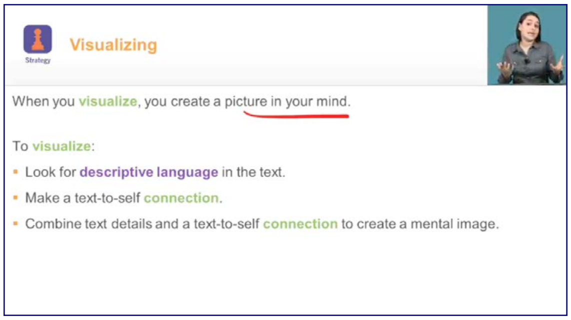


Figure 90. *Grades 6–12 students learn to visualize when reading.*

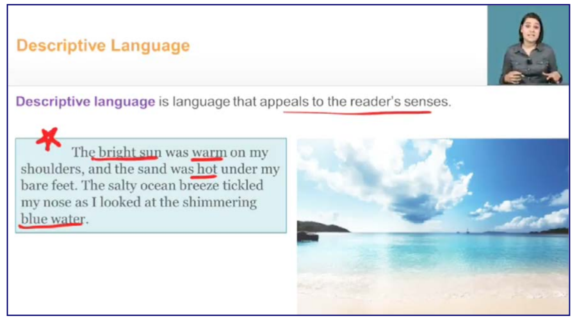


Figure 91. *Grades 6–12 students analyze descriptive language.*

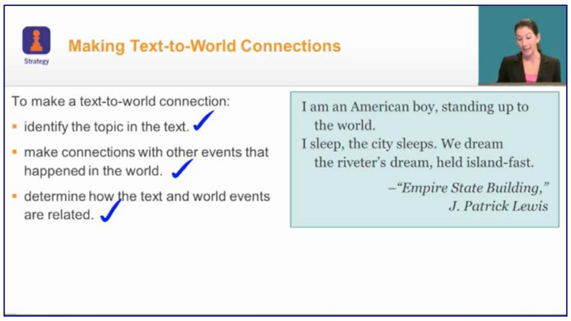


Figure 92. *Grades 6–12 students make text-to-world connections.*

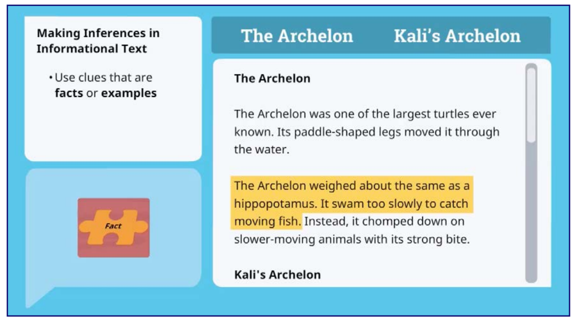


Figure 93. *Grades K–5 students use clues to make inferences.*

Lessons that teach **after-reading strategies** focus on identifying the main idea, answering questions, drawing conclusions, summarizing, comparing, and synthesizing. Imagine MyPath recognizes that summarizing is a challenging skill for many students. Therefore, students have ongoing opportunities to practice this. In an interactive Grades K–5 lesson, students identify and summarize the main idea of the story by dragging and dropping the sentences in the order in which they occurred (Figure 94). In a Grades 6–12 lesson, students are taught how to analyze a passage and draw conclusions from a story based on an author's point of view (Figure 95). Students consider whose thoughts and feelings the narrator reveals to practice drawing conclusions using evidence from the text.

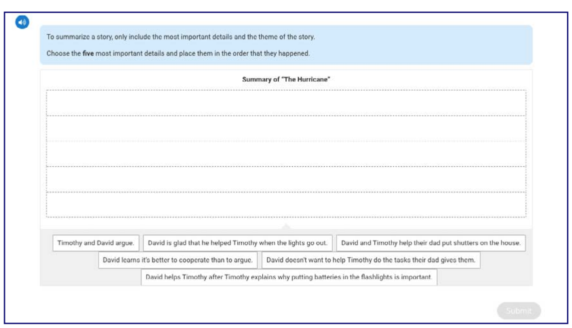


Figure 94. *Grades K–5 students summarize the main idea of a story.*

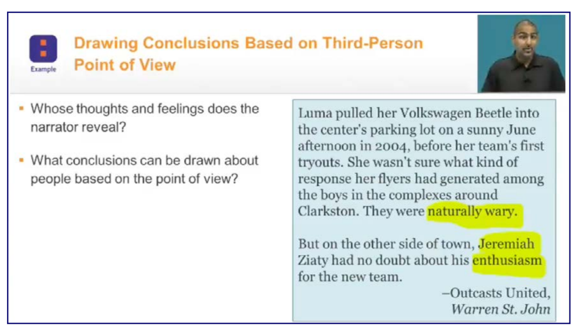


Figure 95. *Grades 6–12 students draw conclusions based on point of view.*

Imagine MyPath ensures all students have access to rigorous, grade-level content by providing opportunities for **close reading of complex texts**. As students progress in their ILP, they read literary and informational texts that increase in complexity and depth. They answer text-dependent questions in which they are expected to explain and justify their responses using specific evidence from the text (e.g., a word, phrase, or passage). Scaffolding, hints, and reteaching are used to support students' close reading and reinforce essential concepts and skills.

Students learn how to analyze *what the text says* by answering *who, what, when, where, why,* and *how questions.* This guides their reading and helps them understand the main ideas and details in the text. For instance, students read informational texts and answer multiple choice questions like, "What is the main idea of 'The Magic of Coral Reefs'?" (Figure 96). They answer similar questions in a Grades 6–12 lesson, also focused on identifying the main idea of a paragraph about bottlenose dolphins (Figure 97).

Students also examine *how the text works*. They explore figurative language, text features, literary and poetic devices, and word choice. In a Grades K–5 lesson, students are provided with descriptions and examples of similes and metaphors in text before reading passages independently (Figure 98). Students practice identifying similes and metaphors in the text (e.g., "The water is a warm blanket, soothing my nerves") and explaining their meaning (e.g., "It means the water is calming to the narrator.") (Figure 99).

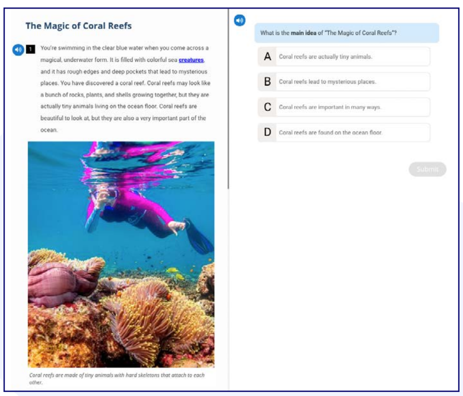


Figure 96. *Grades K–5 students answer text-dependent questions.*

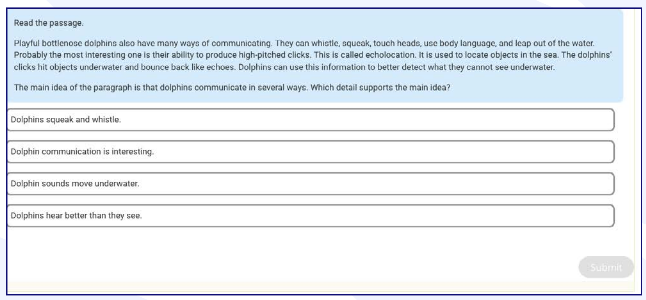


Figure 97. *Grades 6–12 students answer text-dependent questions.*

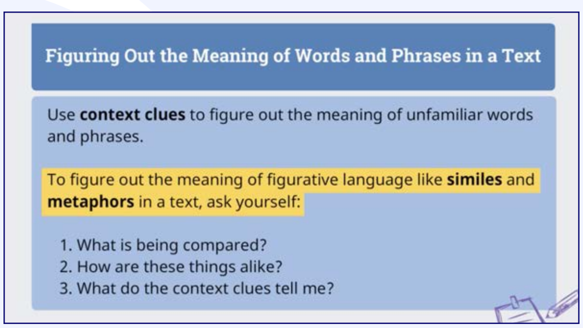


Figure 98. *Grades K–5 students explore figurative language.*

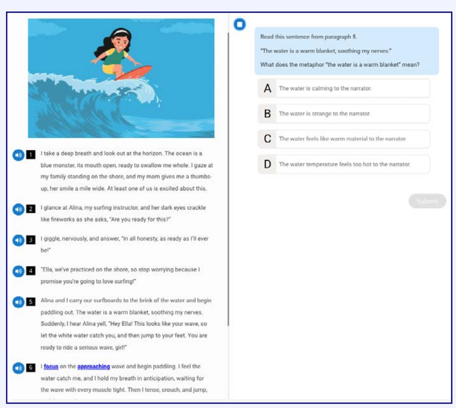


Figure 99. *Grades K–5 students identify similes and metaphors.*

Students reflect on *what the text means* by answering questions that concentrate on an author's purpose (stated or implied), synthesizing, and opinion writing. For instance, when learning about an author's purpose, the teacher explains that every author has a purpose or reason for writing, such as entertaining, informing, or persuading the reader to agree with what the author says (Figure 100). In a Grades K–5 lesson, students read three short texts with scaffolding (e.g., metacognitive bubbles, text highlighting) to help them attend to certain details that signify the author's purpose (Figure 101).

When reflecting on what the text means, students are also taught how to synthesize information. In a Grades 6–12 lesson, students compare and contrast ideas across two informational texts (Figure 102). Students are prompted to

> "Read each text and analyze them individually. What are they about? What are the key details? Second, compare and contrast the articles. How are they the same? How are they different? Third, consider what you already know about the topic or how the topic makes you feel. Finally, combine information and making conclusion based on the details."

In another lesson, students use evidence to form an opinion (Figure 103). They read a biography about Frida Kahlo, which contains subjective language, and practice forming an opinion that is backed with evidence (e.g., citing details from the text best supports the key idea that Frida Kahlo was a confident woman) (Figure 104).

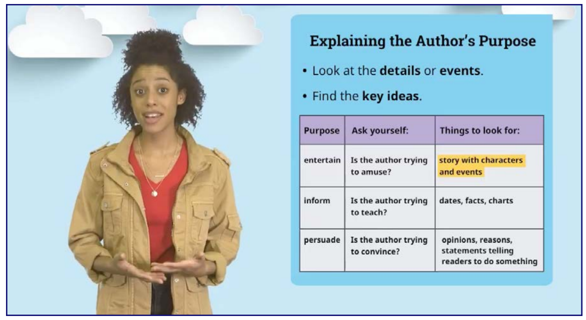


Figure 100. *Grades K–5 students learn about author's purpose.*

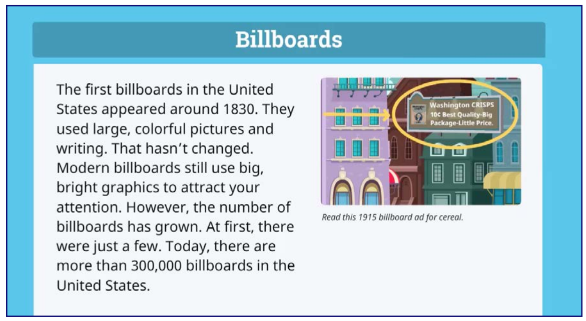


Figure 101. *Grades K–5 students determine the author's purpose.*

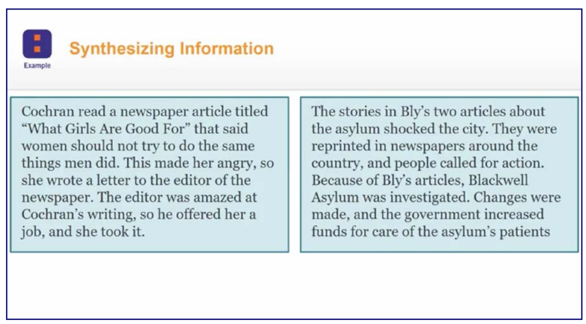


Figure 102. *Grades 6–12 students synthesize information across texts.*

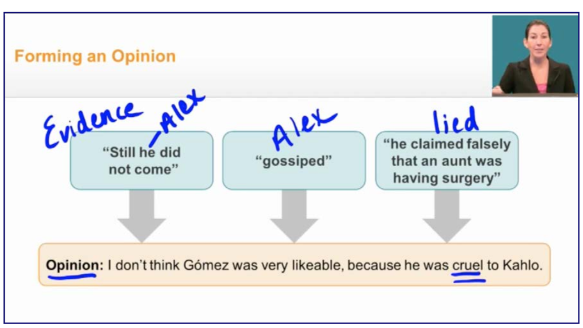


Figure 103. *Grades 6–12 students learn how to form an opinion.*

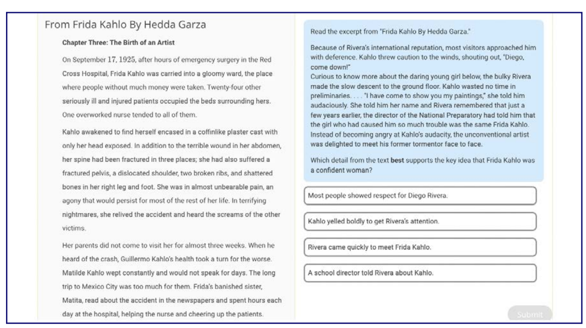


Figure 104. *Grades 6–12 students use evidence to form an opinion.*

Principle 4. Incorporate Evidence-Based Practices for Teaching Mathematics

Preparing students to think and reason mathematically is essential for modern society and our global economy (Boaler, 2016; Clements & Sarama, 2021). Mathematics plays a critical role in students' academic success and future career opportunities (National Research Council [NRC], 2012). Studies show that students' early mathematics knowledge predicts their success in algebra and overall mathematics achievement in high school (Aunio et al., 2015; Baroody & Purpura, 2017; Clerkin & Gilligan, 2018; Powell & Fuchs, 2012; Siegler et al., 2012). Therefore, educators are doubling down on their efforts to ensure instruction adequately supports students' conceptual understanding and procedural fluency.

Developing a Conceptual Understanding

Conceptual understanding (comprehension and connection of concepts, operations, and relations) establishes the foundation for developing procedural fluency (meaningful and flexible use of procedures to solve problems) (National Council of Teachers of Mathematics [NCTM], 2014). Building relationships between conceptual knowledge and procedures for problem solving is advantageous for understanding mathematics (Crooks & Alibali, 2014; Hiebert & Lefevre, 2013; Rittle-Johnson et al., 2015). Emphasis has been placed on pedagogy that promotes quantitative reasoning; perseverance when problem solving; mathematical modeling; the strategic use of tools; accuracy and precision; making use of structure; looking for patterns in reasoning; and recognizing the practicality of mathematics (National Governors Association of Best Practices, 2010; NCTM, 2014).

The **concrete-representational-abstract (CRA)** framework is a well-documented approach that helps students understand the conceptual underpinnings of concepts taught using models and representations (Agrawal & Morin, 2016; Bruner & Kenney, 1965). It has demonstrated success across mathematical domains, grade levels, and skill levels (Flores et al., 2018; Root et al., 2021). In the concrete phase, teachers use 3-dimensional manipulatives (e.g., base-10 blocks, fraction circles, algebra tiles) to create a mental model of the concept. In the representational phase, teachers use 2-dimensional pictures (e.g., number lines, bar models, diagrams, graphs, or drawings) to demonstrate the same concept. In the abstract phase, students apply this conceptual knowledge to solve mathematics problems symbolically.

Relatedly, when teachers connect mathematics to students' lives, students see how it can apply to their own life. Grounding mathematics in **real-world contexts** that are relevant to students (Ladson-Billings, 2009) is a powerful tool for promoting problem solving and conceptual understanding (NCTM, 2014). Researchers agree that effective mathematics instruction incorporates opportunities for students to apply their understanding across disciplines and to their daily lives (Matthews, 2018; Ottmar et al., 2015; Walkowiak et al., 2014).

Building Procedural Fluency

Much like conceptual understanding, **procedural fluency** is important in its own right; students need opportunities to develop flexibility in their approach to solving problems (Bay-Williams, 2020; NRC, 2012). Students develop procedural knowledge as they learn that symbols represent ideas and that certain rules, algorithms, and procedures can be applied to a given situation (Hiebert & Lefevre, 2013). This is important because procedural fluency impacts students' work with whole-number operations, fractions, geometry, measurement, and algebra (Huinker & Bill, 2017; National Mathematics Advisory Panel, 2008). However, if rules and procedures are taught prematurely, students can have trouble understanding why a procedure works (Bay-Williams, 2020). This can lead to confusion, misconceptions, and the misuse of strategies (Rittle-Johnson et al., 2001). Experts recommend students have adequate time to develop a conceptual understanding of concepts underlying procedures (Star et al., 2015). Instruction should incorporate visual representations and models to help students learn and select effective strategies to solve problems; reflect on and explain why a rule, algorithm, or procedure works; and make explicit connections between concepts and procedures (Bay-Williams, 2020).

Prioritizing Essential Skills

Students' academic success requires a deep understanding of foundational mathematics concepts. Notably, students' early knowledge lays the foundation for their work with increasingly sophisticated concepts in later grades (Baroody & Purpura, 2017; Powell & Fuchs, 2012; Siegler et al., 2012). Prioritizing grade-level standards that emphasize depth of learning, not breadth, is widely accepted and proven to be more effective than covering every concept or skill (Ainsworth, 2013). Instruction should take learning progressions into account to determine what students must be able to do by the end of the year, in preparation for what students will learn next (Ainsworth & Donovan, 2019). Research suggests that progressions prioritize the following domains and the essential concepts and skills that underpin them—number and operations, algebra, measurement and data, and geometric reasoning (NRC, 2012; NCTM, 2014).

Number and Operation

Number Sense. Number sense is foundational for developing mathematics proficiency (Clements & Sarama, 2021; Hannula-Sormunen et al., 2015; Jordan & Dyson, 2016). Number sense involves thinking flexibly about numbers (approximation and estimation), accurately assessing numerical magnitude (comparison and equivalence), and understanding numerical writing conventions and place-value concepts (Witzel et al., 2013). There is empirical evidence that shows students' early number sense predicts their fluency with basic facts (Jordan et al., 2010), understanding of part-part-whole relationships, and later mathematics achievement (Nguyen et al., 2016). This is important because the "primary cause of problems with the basic combinations, especially among children at risk for or already experiencing learning difficulties, is the lack of opportunity to develop number sense" (Baroody et al., 2009, p. 69). To ensure students have opportunities to develop a strong understanding of number, the following concepts should be taught at each level of the developmental progression (Clements & Sarama, 2021; Frye et al., 2013; Jordan & Dyson, 2016):

- ***Counting:*** Provide a strong focus on counting (assigning a number to a set or group of objects), one-to-one correspondence (each counted number refers to an item in the set), and how counting relates to cardinality (the last number in the count is the value for the set) (Clements & Sarama, 2021; Fuchs et al., 2013).
- ***Number after knowledge:*** Teach students how to count on from numbers other than one to support efficient counting principles (Fuchs et al., 2013).
- ***Number comparison:*** Promote magnitude understanding of numbers using a range of concrete models (Siegler et al., 2011). Teach students how numbers are represented by printed numerals (Sarama & Clements, 2009) and incorporate number line representations to help them develop a mental number line (Scalise & Ramani, 2021; Siegler, 2016).
- ***Place value:*** Tailor instruction around helping students understand the structure of the base-10 system, the relationship between the places, and what it means to unitize (grouping objects into a smaller set) (Brendefur et al., 2018; Hartnett, 2018; MacDonald et al., 2018).
- ***Estimation:*** Teach number line estimation strategies to improve arithmetic performance (Maertens et al., 2016). Build middle and high school students' number sense by providing opportunities that encourage students to practice mental and computational estimation (Groth, 2013).

How Imagine MyPath Helps Students Develop Number Sense

Imagine MyPath helps students develop **number sense** by explicitly teaching essential concepts at each level of the developmental progression. Students learn **counting strategies** by exploring everyday objects. Lessons emphasize one-to-one correspondence (students count each number) and the cardinality principle (restating the last counted word, "4 shells," to reinforce how many total shells are in the set) (Figure 105). Students learn strategies like counting on from numbers other than one. They also learn that it is more efficient to count on from the larger number to find the total. In Figure 106, students apply this strategy as they hold up three fingers to represent the larger addend (3) and count on "4…5." Students practice **comparing adjacent numbers** mentally and symbolically, beginning with smaller quantities before comparing, ordering, and estimating larger quantities. For example, the teacher says, "Each time I add one more to a number, I can rename the next number in the counting order." Students see a range of examples with concrete models and numbers (Figure 107) and hear mathematics vocabulary (e.g., "there are *fewer* jump ropes than hoops," "6 is *greater* than 5") to emphasize magnitude understanding (Figure 108).

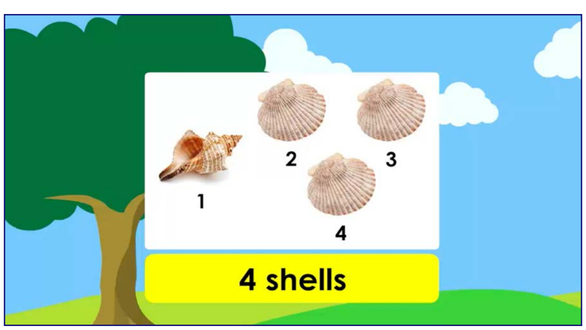


Figure 105. *Grades K–5 students count common objects.*

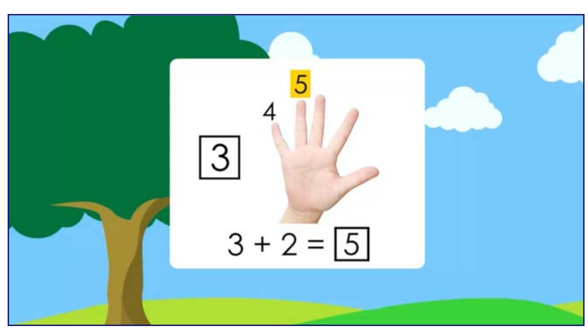


Figure 106. *Grades K–5 students use counting-on strategies.*

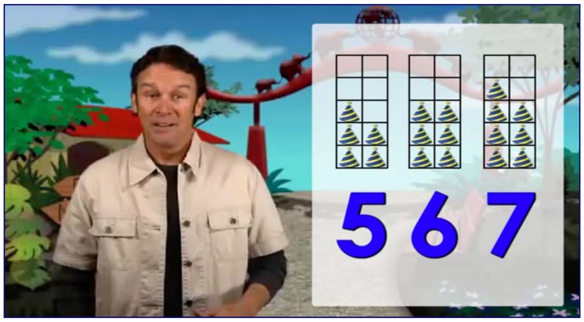


Figure 107. *Grades K–5 students learn number-after principles.*

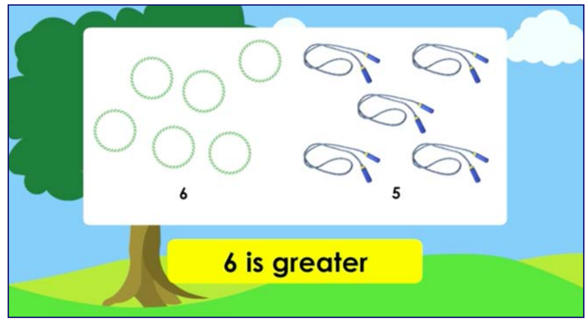


Figure 108. *Grades K–5 lessons reinforce mathematics vocabulary.*

Imagine MyPath integrates concrete and visual representations to help students understand **place value** and the base-10 system. In a Grades K–5 lesson that emphasizes unitizing, students learn how to bundle items, such as crayons, into groups of tens and ones (Figure 109). The program explains that the digit in the tens place indicates how many groups of ten there are, while the digit in the ones place indicates how many ones. Students compare whole numbers using base-10 blocks, number sentences, and language (e.g., “less than,” “equal to,” “greater than”) (Figure 110). In a Grades 6–12 lesson, students explore place value to the millions (Figure 111). The teacher integrates multiple representations to help students build a conceptual understanding of place value. The teacher also models how to decompose a three-digit number and regroup using base-10 blocks (Figure 112).

Figure 109. *Grades K–5 lessons introduce unitizing principles.*

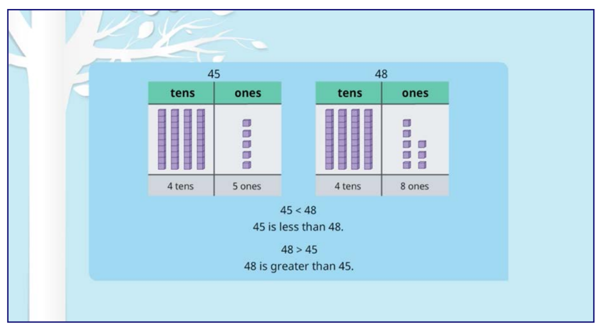


Figure 110. *Grades K–5 lessons use multiple representations.*

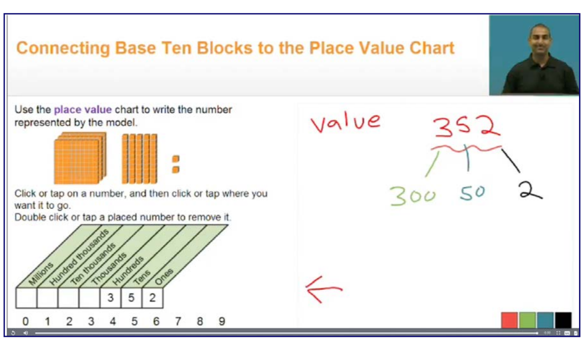


Figure 111. *Grades 6–12 lessons use multiple representations.*

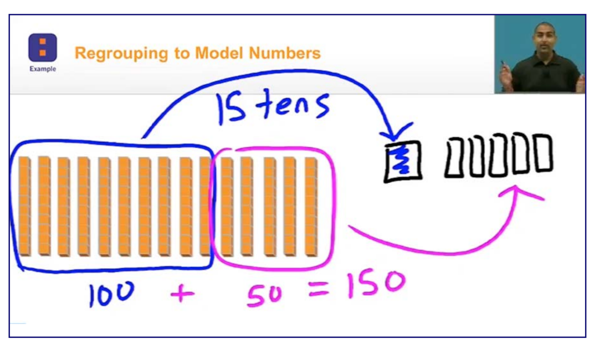


Figure 112. *Grades 6–12 teachers regroup multidigit numbers with models.*

To support students' number sense development, Imagine MyPath incorporates a variety of pictures, models, representations, real-world scenarios, and video demonstrations to help students develop **estimation strategies.** In a Grades K–5 lesson, students apply estimation strategies as they solve word problems involving length (Figure 113). Students use standard (feet) and nonstandard units (sheets of paper) to estimate. Students are also encouraged to reason about the unit they choose to measure an object. For instance, estimating in smaller units, like centimeters, would not be the most efficient or accurate way to measure a conveyor belt (Figure 114). In another lesson, students in Grades 6–12 apply estimation strategies to solve computation problems and build fluency. The teacher models how to use estimation strategies to compare square roots (Figure 115). Students solve problems in which they estimate and assess the reasonableness of their answer, such as "Which two consecutive whole numbers does √39 lie between? Why?" They also practice estimating and plotting the square root on a number line to reinforce number magnitude knowledge (Figure 116).

Figure 113. *Grades K–5 students learn estimation strategies.*

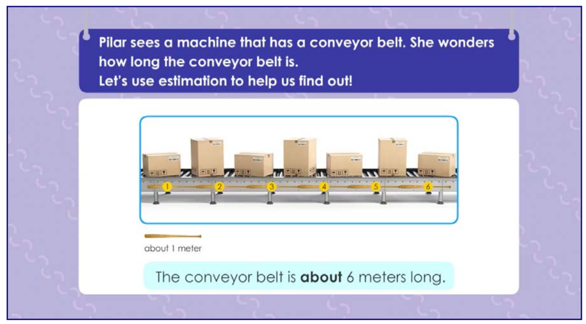


Figure 114. *Grades K–5 students select proper units to measure objects.*

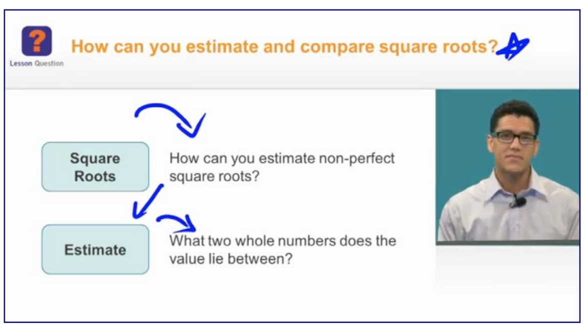


Figure 115. *Grades 6–12 students solve problems using estimation strategies.*

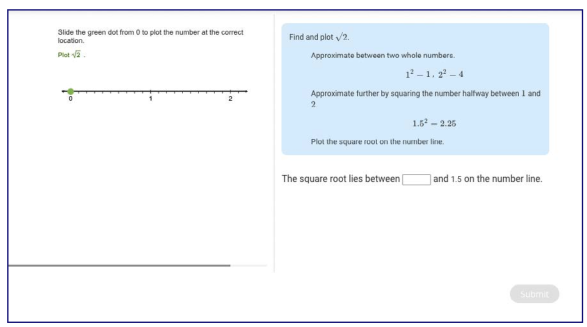


Figure 116. *Grades 6–12 students apply number line estimation strategies.*

Whole-Number Operations. A conceptual understanding of whole-number operations provides a foundation for work with more advanced mathematics concepts (Price et al., 2013). Early mathematics instruction focuses on additive reasoning skills, or students' understanding of part-whole relations (Vergnaud, 1982). Research has found that a strong understanding of part-whole relations and fluency with single-digit **addition** and **subtraction** facts relates to students' proficiency with multidigit addition and subtraction (Hickendorff et al., 2019). As students approach third grade, instruction shifts toward developing students' multiplicative reasoning skills, or the ability to reason about the relationship between two quantities simultaneously (Vergnaud, 1982). This transition is important because multiplicative reasoning is a necessary precursor to proficiency with **multiplication**, **division**, ratio, rate, fractions, and algebra (Askew, 2018;

Downton & Sullivan, 2017; Ebby et al., 2021; Gurganus, 2017; Malola et al., 2020). Researchers caution against instruction that teaches students to memorize procedures before they understand the concepts that underly them, as this can lead to difficulties determining accurate solutions, effective strategies, and communicating their thinking (Baker & Cuevas, 2018). To build conceptual understanding and procedural fluency with whole numbers, effective instruction should:

- Incorporate the CRA framework to help students master whole-number operations (Agrawal & Morin, 2016; Bouck et al., 2018; Flores, 2010).
- Promote real-world application by incorporating a variety of word problem types to promote flexibility in problem solving (Carpenter et al., 2015).
- Encourage multiple strategies, such as the use of manipulatives, representations, and reasoning strategies (Clements & Sarama, 2021; Van de Walle et al., 2018b).
- Teach whole numbers as composite units to support students' shift from additive to multiplicative reasoning (Lamon, 2012; Tzur et al., 2017). This requires ongoing practice composing and decomposing numbers to support students' understanding of multiplicative relations.

How Imagine MyPath Supports Students' Understanding of Whole Numbers

Imagine MyPath incorporates the CRA framework to help students develop a conceptual understanding of mathematics. For example, when students in Grades K–5 learn how to solve problems involving multidigit **addition**, they begin by using concrete models (e.g., base-10 blocks, Figure 117), before being introduced to visual representations (e.g., number line, Figure 118). As students develop a more abstract understanding of the operation, they solve problems using the standard algorithm (Figure 119).

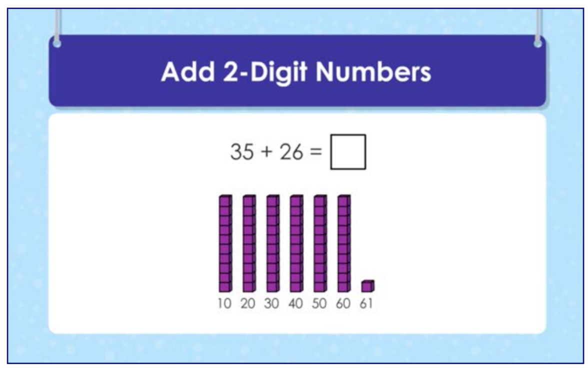


Figure 117. *Grades K–5 students use concrete models to add 2-digit numbers.*

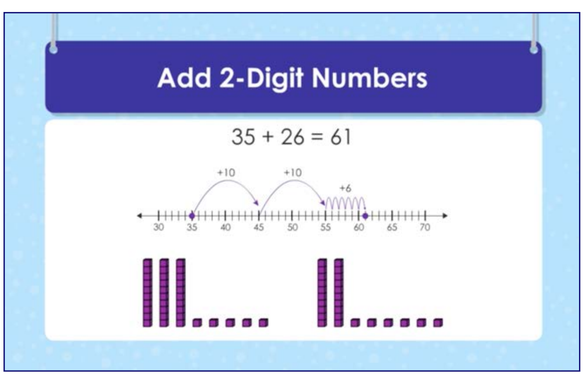


Figure 118. *Grades K–5 students use number lines to solve problems.*

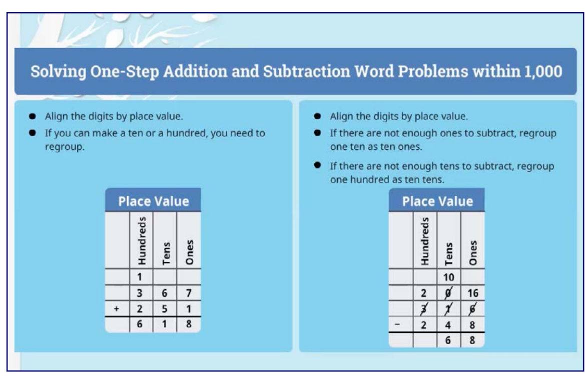


Figure 119. *Grades K–5 students solve problems using the standard algorithm.*

Lessons use real-world contexts to promote conceptual knowledge, procedural skills, and problem-solving strategies. This allows students to apply mathematics to their lives. Students are introduced to a variety of word problem types to encourage flexibility and efficiency when determining a strategy to use to solve an **addition**, **subtraction**, **multiplication**, or **division** problem. Figure 120 provides an example of a Grades K–5 addition problem with a missing addend, while Figure 121 displays an example of a Grades K–5 compare problem type. In Figure 122, students in Grades 6–12 solve a partitive division problem. Exposure to problems beyond the traditional addition and subtraction problems in which the result is unknown helps students understand the underlying structure of word problems and strengthens their flexibility when problem solving.

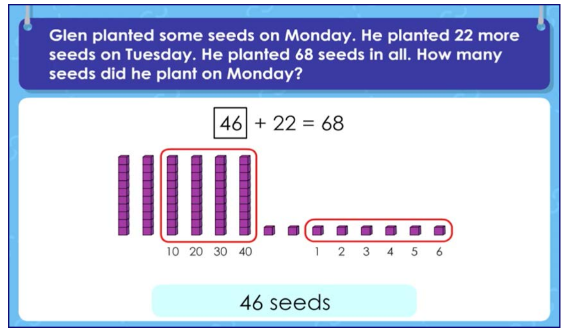


Figure 120. *Grades K–5 students solve word problems with a missing addend.*

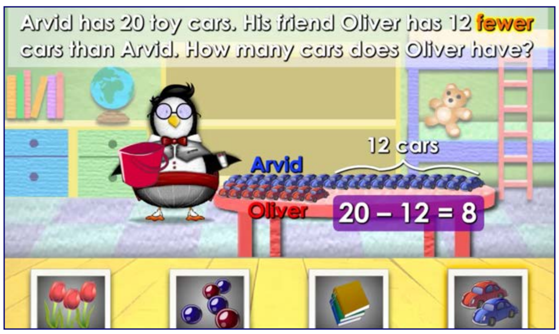


Figure 121. *Grades K–5 students solve a compare word problem.*

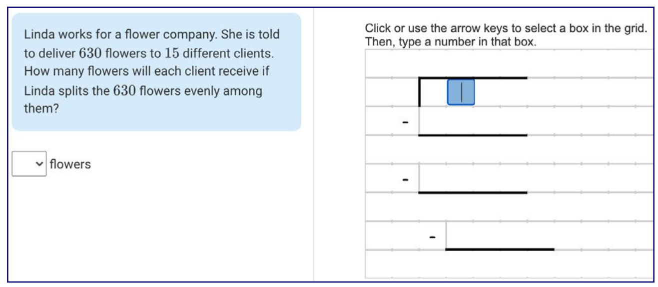


Figure 122. *Grades 6–12 students solve a partitive division word problem.*

Students are taught to use a range of strategies when solving problems (e.g., models, representations, reasoning and estimation strategies, standard algorithms). For instance, in a Grades 6–12 lesson, students first learn how to solve whole-number **multiplication** and **division** problems using models (base-10 blocks, Figure 123). Physically representing the process helps students develop a deeper understanding of the operation. Then, students are taught how to use representations (e.g., area models) because these are closely related to the computation involved with using the standard algorithm. In another Grades 6–12 lesson, students apply their understanding of the operation and standard algorithm by analyzing a worked example (Figure 124). They also learn reasoning strategies to build efficiency when

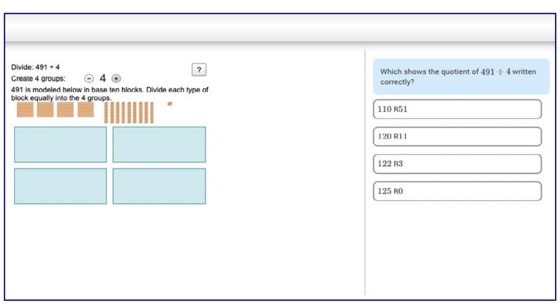


Figure 123. *Grades 6–12 students use base-10 blocks to divide.*

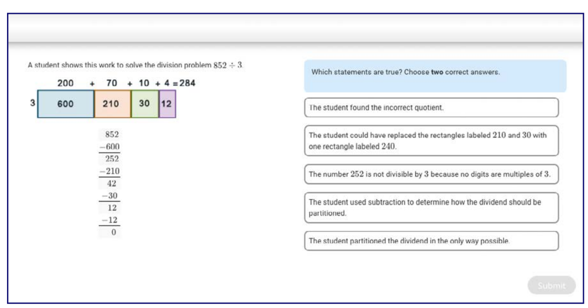


Figure 124. *Grades 6–12 students analyze a worked example.*

solving problems. For instance, they respond to the following problem: "I bought six presents that ranged between $35–$42 in price. Which would be a reasonable amount I would spend on all those presents?" Students are encouraged to evaluate the reasonableness of their answer to see if it is close to what the exact answer should be.

Imagine MyPath emphasizes the connections between **addition** and **multiplication** across a range of topics to promote students' transition from additive to multiplicative reasoning. In a Grades K–5 lesson, students learn how the word problem could be represented using repeated addition and multiplication (Figure 125). In a geometry lesson, students learn that adding all the sides of a pentagon together (repeated addition) is equivalent to multiplying one side's length by the number of sides (Figure 126). This emphasizes whole numbers as composite units. Arrays are also used to promote a conceptual understanding of multiplication and build fluency with basic facts. Students learn how the array changes with different groupings and that the order of multiplication does not matter (Figure 127).

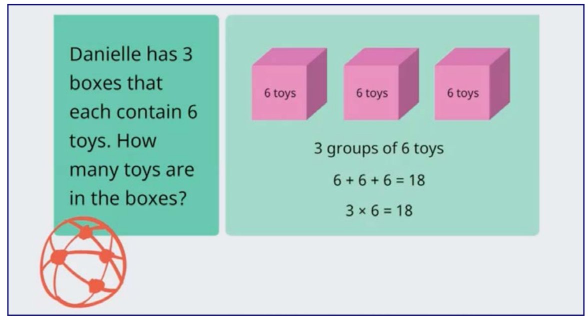


Figure 125. *Grades K–5 students use repeated addition and multiplication.*

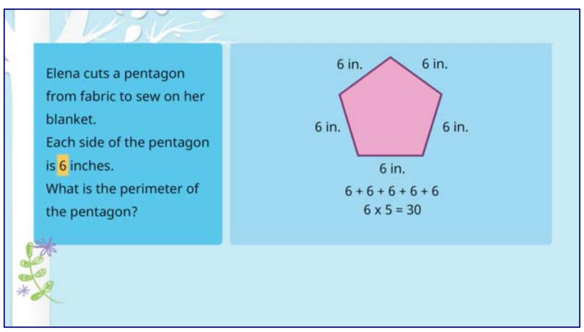


Figure 126. *Grades K–5 students find the perimeter of a pentagon.*

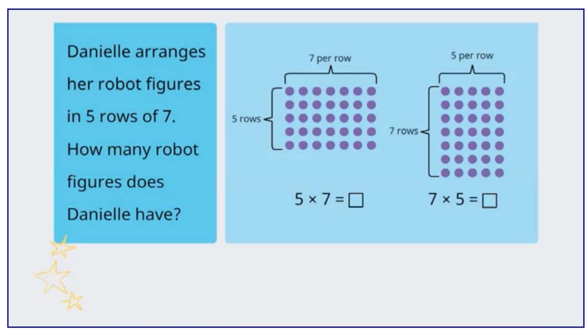


Figure 127. *Grades K–5 lessons reinforce the commutative property.*

Rational Numbers. Many researchers regard the rational number system as the bedrock for students' work with fraction operations, decimals, percentages, and algebra (Bailey et al., 2012; DeWolf et al., 2015) and overall success in mathematics (Booth et al., 2014; McMullen & Van Hoof, 2020). A conceptual understanding of rational numbers "requires understanding the multiple interpretations of rational numbers, skill at translating among the three notations [fractions, decimals, percentages], and knowledge of when each numerical notation is most convenient to us" (Tian & Siegler, 2018, p. 353). Yet, these are considered a pervasive and persistent source of difficulty (Hunt et al., 2018; Tian & Siegler, 2018). Many students lack a deep understanding of fraction and decimal concepts (Kainulainen et al., 2017), misapply whole-number principles (Malone & Fuchs, 2017; Namkung et al., 2018), do not learn how to effectively use models and representations to make sense of rational number concepts, or rely too heavily on algorithms (Flores et al., 2019). Researchers have advocated for a stronger emphasis on rational number instruction to help students develop proficiency and fluency with rational number operations (DeWolf et al., 2015;

Lortie-Forgues et al., 2015). Research supports the following instructional strategies to improve students' understanding of **fractions, decimals,** and **percentages**:

- Teach students that fractions can be interpreted and represented in a variety of ways (Groth, 2013; Van de Walle et al., 2018b). Use a range of visual representations (e.g., part-whole models, number lines) to explore fraction equivalence and comparisons (Lamon, 2012; Siegler et al., 2010).
- Use knowledge of whole number place value, fractions, and place-value labels (e.g., read 0.34 as "34 hundredths" rather than "point 34") to write and compare decimals as fractions to boost students' decimal magnitude knowledge and their relationship to common fractions (Malone et al., 2019).
- Teach conceptual strategies (e.g., finding a common unit) before introducing algorithms (e.g., invert and multiply) to emphasize why the procedures make sense (Cramer et al., 2010). This also prevents misapplication of whole-number principles (Fuchs et al., 2017; Lamon, 2012; Siegler et al., 2010).
- Support students' understanding of percentages as a proportional relationship by using models (grid models, comparison scales, ratio tables, bar models, double number lines), presenting different types of word problems (e.g., percent increase, percent decrease), and providing opportunities to translate across notations (percentages, fractions, decimals) (Mula & Hodnik, 2020).

How Imagine MyPath Supports Students' Understanding of Rational Numbers

The program helps students develop a strong understanding of rational numbers by emphasizing fundamental concepts like unit fractions, partitioning, and part-whole relations. To reinforce the concept that fractions can have multiple meanings, lessons across the curriculum illustrate how fractions can be used to show different interpretations (part-whole, quotient, measurement, ratio, and operator). For instance, a Grades K–5 lesson incorporates a real-world problem that requires students to interpret a fraction as division (quotient interpretation) (Figure 128). In another Grades 6–12 lesson, students learn to interpret fractions as a ratio (Figure 129), which connects to another important skill—proportional reasoning.

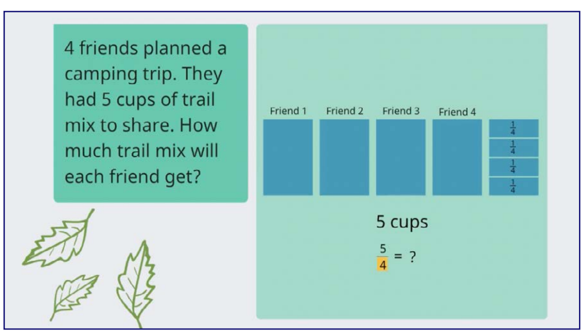


Figure 128. *Grades K–5 students interpret a fraction as division using models.*

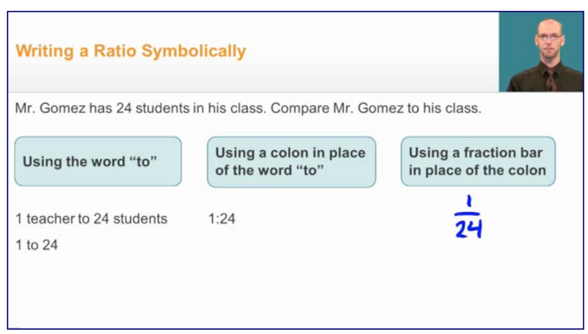


Figure 129. *Grades 6–12 students interpret a fraction as a ratio.*

Imagine MyPath recognizes that students' **fraction-magnitude understanding** can influence their proficiency with fraction operations. Therefore, lessons intentionally focus on comparing, ordering, and finding equivalent fractions. In a Grades K–5 lesson, students read a word problem about running a mile. The teacher uses a part-whole diagram to represent a mile and explains, "The mile is divided into eight equal parts and five of those eight equal parts shows five-eighths of a mile" (Figure 130). Then, the teacher likens this visual model to the number line to show the equivalent amount. In a Grades 6–12 lesson, students build on this understanding by comparing equivalent fractions using representations and expressions (Figure 131). Students are taught to reason about the size of the parts when comparing fractions, an important strategy for building conceptual understanding.

To emphasize the connection between fractions and **decimals**, Grades K–5 lessons teach students how to write decimal fractions (a fraction with a denominator of 10, 100, 1,000, etc.) using place-value charts and decimal grids (Figure 132). Lessons also model precision when discussing decimals ("one tenth is equivalent to ten hundredths") to avoid misconceptions. In Grades 6–12 lessons, students apply their understanding of decimals as they solve real-world problems that require them to compare decimals (Figure 133) and later multiply and divide by powers of ten (Figure 134).

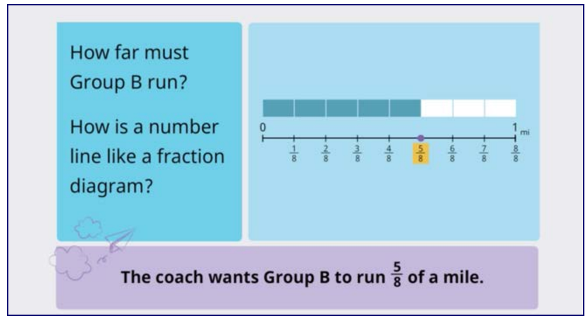


Figure 130. *Grades K–5 students explore fractions using models.*

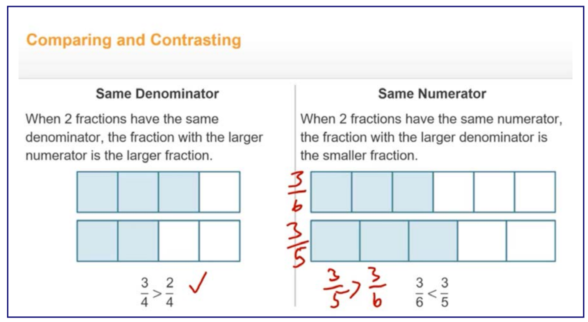


Figure 131. *Grades 6–12 students compare fractions.*

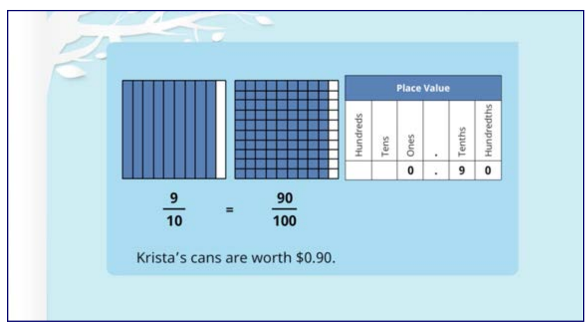


Figure 132. *Grades K–5 students read and write decimals.*

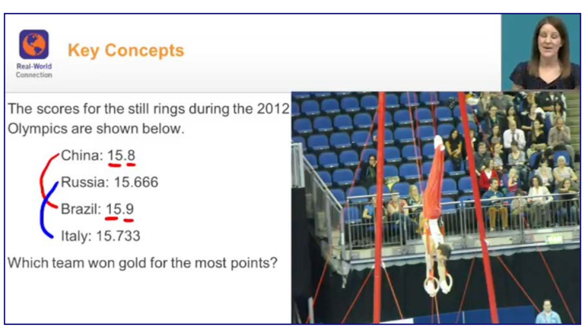


Figure 133. *Grades 6–12 students solve real-world problems with decimals.*

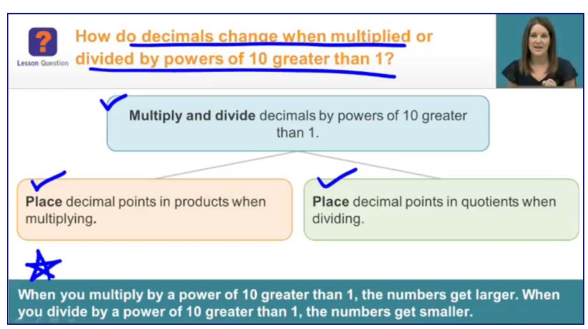


Figure 134. *Grades 6–12 lessons emphasize powers of ten.*

To promote proficiency with **rational number operations**, Imagine MyPath encourages conceptual strategies before introducing algorithms. While adding and subtracting fractions is typically easier for students, multiplying and dividing fractions is often a source of struggle. Lessons help students make sense of the operations using real-world scenarios and visual models (Figures 135 and 136). In Figure 136, the teacher explains, “To divide the flour into four equal-sized bowls, we need to divide this fourth into three equal pieces. The whole model is divided into 12 equal parts, so each part is 1/12 of the whole.” This helps students understand the process of division conceptually.

Students also learn to translate across notations (fractions, decimals, percentages). In a Grades 6–12 lesson, the teacher models the problem and provides explicit instruction on how to convert both decimals and fractions into the same form (Figure 137). In another lesson, students practice finding the **percentage** change by using the ratio of change in quantity to original amount (“A town’s population increased from 14,523 to 16,489. What is the percent increase in the town’s population?”). Students use bar models to represent the problem and deepen their conceptual understanding before applying a formula (Figure 138). Not only do percentage increase and decrease problems help students conceptualize changes in value over time, but they are also highly relevant to their daily lives.

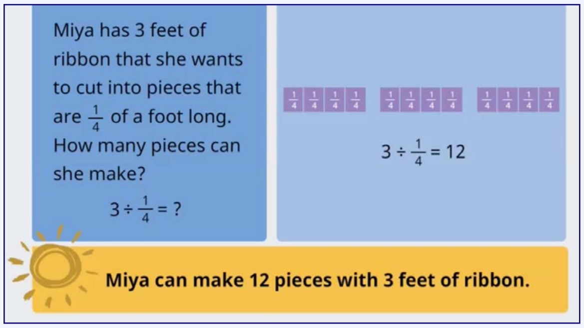


Figure 135. *Grades K–5 students use models to support fraction operations.*

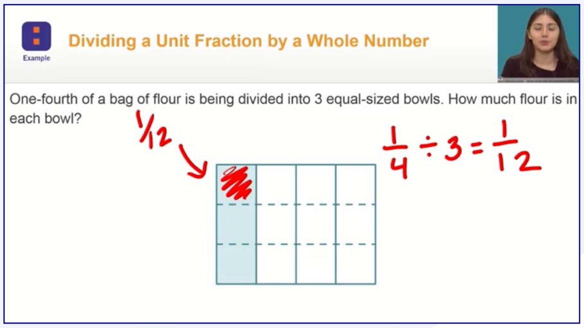


Figure 136. *Grades 6–12 students use models to support fraction operations.*

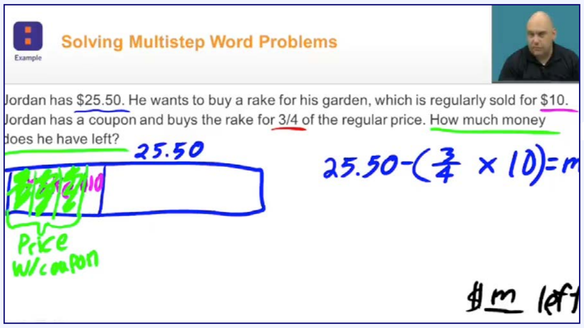


Figure 137. *Grades 6–12 students translate across notations.*

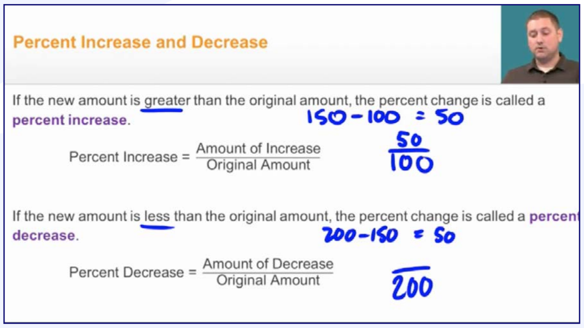


Figure 138. *Grades 6–12 students solve percentage change problems.*

Ratios and Proportional Reasoning. Proportional reasoning is "mathematical reasoning involving a sense of co-variation and the ability to make multiple comparisons in relative terms" (Hilton et al., 2016, p. 194). In Grades 6–7, students are introduced to concepts of **rate, ratio,** and **proportions** (National Governor's Association Center for Best Practices, 2010). These concepts build on students' previous knowledge of multiplication, division, fractions, decimals, measurement, and scale (Boyer & Levine, 2012); they also require students to draw on their understanding of multiplicative relationships to solve problems involving proportional reasoning (Hunt & Vasquez, 2014). However, many students struggle to develop proportional reasoning and misapply additive reasoning to multiplicative situations, have difficulty determining the difference between proportional and nonproportional relationships, and inappropriately apply algorithms when solving rate and ratio problems (Van Dooren et al., 2005). To support students' conceptual understanding of rate, ratios, and proportions, instruction should:

- Provide multiplicative comparison problems to encourage students' use of proportional reasoning (Brown et al., 2020).
- Teach students to attend to the problem structure and learn how to accurately translate that information into a drawing representing the multiplicative situation (Jitendra et al., 2013).
- Present a range of contexts where students apply multiplicative reasoning, use a variety of representations to explore proportional relationships, and distinguish between proportional and nonproportional relationships (Hilton et al., 2016; Witzel & Little, 2016).

How Imagine MyPath Helps Students Understand Ratios and Develop Proportional Reasoning Skills

Imagine MyPath integrates multiplicative comparison problems throughout the curriculum to encourage students to use **proportional reasoning** skills. Explicit instruction is used to help students identify which quantity is being multiplied and which number tells how many times (Figures 139 and 140). Students learn to compare equivalent ratios and solve proportions in a variety of contexts before they are introduced to rules and formulas. This ensures students have time to explore multiplicative problem structures and how to accurately represent problems using a model or representation.

Figure 139. *Grades K–5 students solve multiplicative comparison problems.*

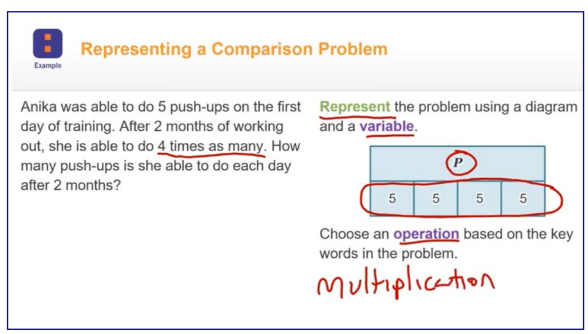


Figure 140. *Grades 6–12 students solve multiplicative comparison problems.*

Imagine MyPath lessons also help students learn to distinguish between a **part-to-part ratio** (a comparison between an amount in one group and an amount in another group) and a **part-to-whole ratio** (a comparison between the amount in one group and the total). Teachers model precise language when describing the relationship between the two quantities, show how ratios can be written three ways (e.g., 2/5, 2:5, 2 to 5), and point out the importance of the order of the numbers in the ratio (Figure 141). Students also explore proportions across mathematical domains. In a Grades 6–12 geometry lesson, students describe how the proportional or nonproportional dimensions of a shape affect other measurements (e.g., surface area or volume) (Figure 142). Students learn how to determine if the corresponding measurements of the rectangular prisms are proportional by creating ratios of the lengths of the corresponding sides. This helps them understand that the ratio of a half is the scale factor of the prism on the left to the prism on the right.

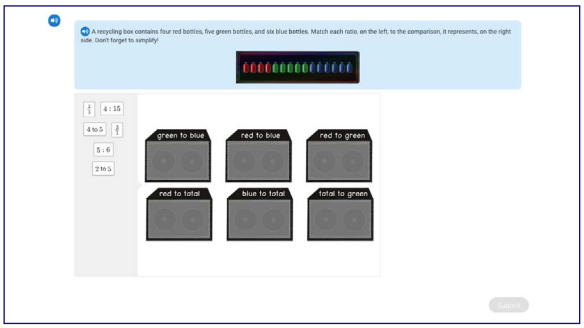


Figure 141. *Grades K–5 students learn to write ratios three different ways.*

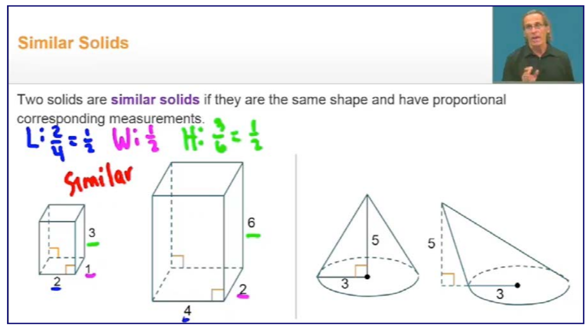


Figure 142. *Grades 6–12 lessons reinforce proportional reasoning.*

Number Systems and Complex Numbers. In middle and high school, students extend their numerical knowledge as they explore the study of **number systems, number theory,** and **complex numbers** (Groth, 2013; National Governor's Association Center for Best Practices, 2010). To do so, they must understand concepts like factors and multiples, properties of operations, divisibility rules, and other number concepts (e.g., prime and composite numbers, rational numbers, and negative numbers) before they explore the complex process involved with irrational numbers (Groth, 2013; Kidron, 2018; Yilmaz & Ay, 2018). Yet, many students struggle to understand irrational numbers conceptually, which impacts their facility with more advanced concepts (e.g., operations with complex numbers) (Groth, 2013; Guven et al., 2011). To address these challenges, research recommends instruction:

- Develop students' understanding of essential concepts (e.g., divisibility rules, prime and composite numbers, negative numbers) (Groth, 2013), integrate examples and non-examples to strengthen numerical knowledge (Zazkis & Leikin, 2007), and use conceptual models (Bofferding, 2018).
- Use number lines (Groth, 2013), geometric representations, and the Pythagorean theorem when introducing irrational numbers (Sirotic & Zazkis, 2007).
- Extend students' work with irrational numbers by having them explore irrational numbers beyond decimal representations or conventional examples like pi and the √2 (Zazkis, 2005).
- Provide opportunities for students to perform arithmetic operations with complex numbers, represent complex numbers and their operations on a complex plane, and solve quadratic equations with real coefficients and complex solutions (Groth, 2013).

How Imagine MyPath Supports Students' Understanding of Number Systems and Complex Numbers

In middle and high school, students have opportunities to apply their understanding and solve more abstract concepts involving **number systems, number theory,** and **complex numbers**. Imagine MyPath Grades 6–12 lessons emphasize essential concepts like divisibility rules, factors and multiples, and negative numbers. Students learn to identify and describe numbers according to their characteristics (factors, multiples, prime, and composite). They use arrays to determine the factors of a number (Figure 143) and explore real-world contexts (e.g., temperatures and elevation) to conceptualize positive and negative numbers (Figure 144).

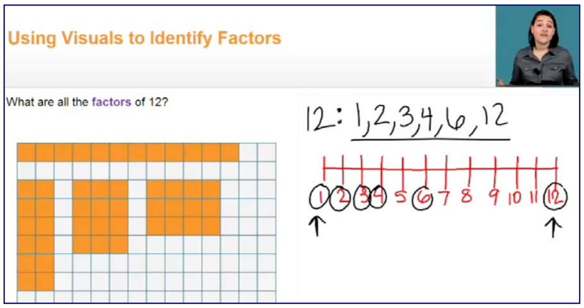


Figure 143. *Grades 6–12 students use visuals to identify factors.*

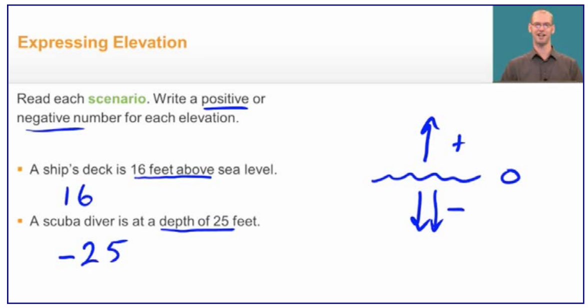


Figure 144. *Real-world contexts teach students about positive and negative numbers.*

Students learn to differentiate between rational and irrational numbers, a challenging concept for many (Figure 145). In a Grades 6–12 lesson, the teacher provides clear definitions and examples of each. They models how to perform operations involving rational and irrational numbers and how to generalize (e.g., rational number × irrational number = irrational number) (Figure 146). Then, students apply this knowledge and practice solving problems like "What is the correct classification of 3/8?" or "Find the sum and then classify it. 5/6 + √91 ."

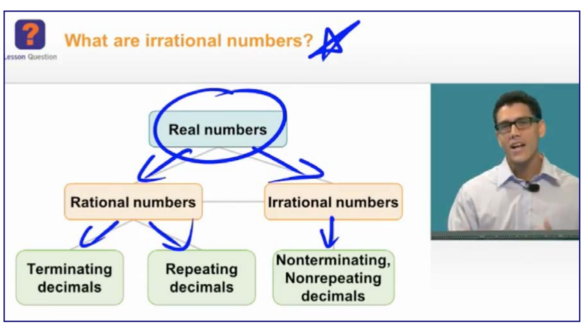


Figure 145. *Grades 6–12 students explore rational and irrational numbers.*

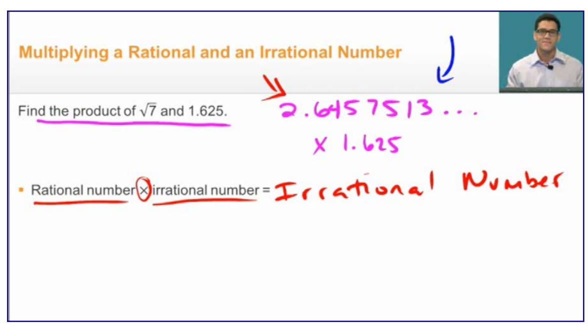


Figure 146. *Grades 6–12 students learn to generalize.*

To prepare students to think in increasingly abstract ways, lessons challenge students to represent complex numbers and their operations on a complex plane, as well as to solve quadratic equations. For instance, students extend their understanding of real and imaginary numbers and practice graphing complex numbers in the plane (Figure 147). Students also apply their understanding of the properties of operations when working with complex numbers. In a Grades 6–12 lesson, the teacher illustrates how the commutative property can be used to write the expression as follows: (3+7i) + (8–6i) as (8+6i) + (3+7i) (Figure 148). Explicit instruction and guided practice are also used to support students' understanding of the process for adding complex numbers.

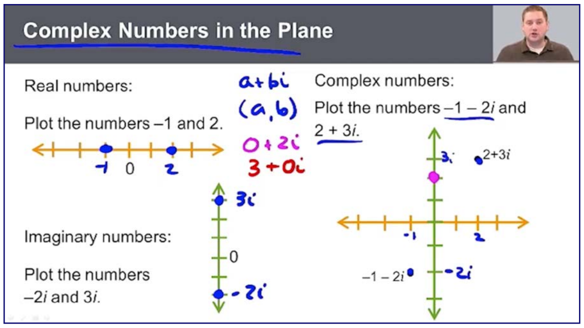


Figure 147. *Grades 6–12 students graph complex numbers in the plane.*

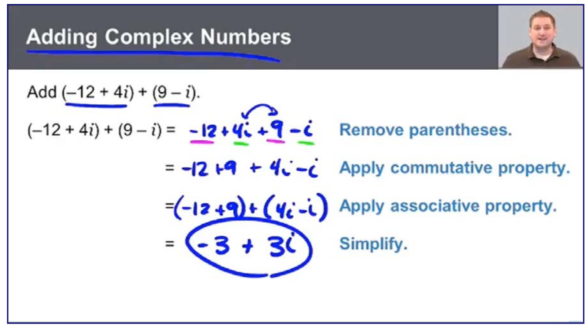


Figure 148. *Grades 6–12 lessons explicitly teach how to add complex numbers.*

Algebra

Algebra is commonly referred to as the "gatekeeper" to school and career success (Groth, 2013). This area of mathematics involves generalized arithmetic, variables, equivalence, understanding and applying knowledge of the properties of operations, solving and simplifying equations, patterns, and functions (Blanton et al., 2019; Groth, 2013; Kaput, 2008; Kieran et al., 2016). Students' experiences attending to structure, reasoning about and representing quantitative relationships, and generalizing promote their use of algebraic thinking (Kaput, 2008; West, 2021). Yet, research has shown that many students develop misconceptions about foundational algebra concepts (Knuth et al., 2016; Welder, 2012). Unaddressed misconceptions can negatively affect problem solving and hinder students' learning new concepts (Booth et al., 2014). Students need exposure to a range of concepts, across grade levels, to understand algebra as an interconnected web of ideas (Blanton et al., 2019; Kaput, 2008; Kieran et al., 2016).

Generalized Arithmetic and Properties of Operations. When students understand arithmetic conceptually and can explain and justify their use of properties of number and operations, students have learned the fundamentals of algebra (Blanton et al., 2015). However, many students lack an understanding of the mathematical structures of operations (Mason, 2016) and do not make connections between the properties used in arithmetic and those used in algebra (Carpenter et al., 2003). To mitigate these challenges, research recommends that instruction focus on helping students apply their understanding of the properties of number and operations to make stronger connections between arithmetic and algebra (Booth et al., 2014; Carpenter et al., 2003).

Interpreting Variables. The ability to understand the meaning behind a variable and how to use variables appropriately is a critical concept in algebra (Blanton et al., 2017; Booth et al., 2014). Students must learn that a variable can be both symbolized and operated on (Wheeler, 1996). Misconceptions appear to increase as students solve more complex equations (Booth et al., 2014; Groth, 2013; Lucariellloa et al., 2014). For instance, students often confuse a variable for a label (Russel et al., 2011). To support students' understanding of a variable and help them learn to accurately translate between words and algebraic notation, students should solve problems using a variety of representations and strategies (Booth et al., 2014; Groth, 2013). They should also solve expressions with one or more variables to reinforce the idea that a variable or symbol can represent an unknown quantity or quantities that vary (Blanton et al., 2015; Stephens et al., 2015).

Equivalence. Many students fail to develop a relational understanding of equivalence (Groth, 2013; Powell et al., 2020). Studies show students who interpret the equal sign as a symbol of action (a command to carry out a calculation), rather than a symbol indicating both sides of the equation are equivalent in value, perform lower on algebra tasks, with more profound negative consequence across grade levels (Byrd et al., 2015). This incorrect view inhibits their ability to flexibly apply strategies and procedures to solve problems. To strengthen students' understanding of equivalence, instruction should use models and representations to teach the relational meaning of the equal sign and contradict the misconception that it is a symbol denoting an action (Faulkner et al., 2016; Powell & Fuchs, 2010).

Solving and Simplifying Expressions, Equations, and Inequalities. Students extend their understanding of arithmetic by solving, simplifying, and writing algebraic equations (Bush & Karp, 2013; Jupri et al., 2014). The transition from solving equations to solving inequalities, simplifying expressions, understanding the order of operations, variables, equivalence, and errors involving the negative sign pose many challenges for students (Booth, 2014; Bush & Karp, 2013; Groth, 2013; Jupri et al., 2014; Seng, 2010). To improve students' computation skills, instruction should:

- Incorporate open number lines and graphs of functions to strengthen understanding and help students make sense of negative numbers (Bush & Karp, 2013).
- Integrate the use of algebra tiles so students can learn how to manipulate variables, apply knowledge of properties of operations, and explain and justify their work (Groth, 2013).
- Provide real-world contexts to help students learn how to simplify expressions and solve equations with radicals and exponents, linear and quadratic equations, and inequalities (e.g., comparing equivalent representations, matching expressions with representations) (Bush & Karp, 2013; Groth, 2013; Swan, 2000; Tsamir et al., 1998).

Patterns and Generalizing. An important goal of algebra is to help students abstract from computations. Students learn to do this as they analyze patterns and learn to generalize (Blanton et al., 2019; Driscoll, 1999). Research has found that recursive and explicit patterns provide rich opportunities for generalizing (Callejo & Zapaterna, 2017; Groth, 2013; Jurdak & Mouhayar, 2014). The underlying structure of a pattern lends itself to reasoning that prompts students' attention to relationships, supports their ability to generalize, and helps them build connections across concepts (West, 2021). These experiences prepare students to reason more flexibly about functional relationships in middle and high school (Ellis, 2011), a pre-algebra skill that is central to students' work in algebra, calculus, and beyond (Bush & Karp, 2013). Research recommends that instruction integrate patterns to encourage students to reason about additive and multiplicative relationships and learn to generalize (e.g., explain and justify a rule) (Knuth et al., 2016; NCTM, 2014; Witzel et al., 2003).

Functions. In algebra, understanding the quantitative relationship between quantities is directly related to the concept of a function (Bush & Karp, 2013). Experts agree students need to develop a conceptual and procedural understanding of functional relationships. Conceptually, students must understand the independent and dependent variables (Kalchman & Koedinger, 2005), and be able to explain the meaning of slope, as well as functional and physical applications (Groth, 2013). Procedurally, students must learn to move flexibly across representations of functions (e.g., equations, graphs, tables) (Warren et al., 2016). Yet, many students struggle to understand what is and is not a function, the relationship between quantities, and how different representations can be used to model a function (Dubinsky et al., 2013). This affects their ability to construct, compare, and interpret linear, quadratic, and exponential models (Graf et al., 2018). To promote a conceptual and procedural understanding of functions, instruction should:

- Integrate real-world scenarios to introduce students to the concept of functions, the notion of covariance and correspondence, and slope (Groth, 2013; Pierce, 2005).
- Provide opportunities for students to explore how functions can be displayed using different, yet complementary, representations (e.g., graphs, tables, equations, verbal explanations) (Groth, 2013; Heid & Blume, 2008; Warren et al., 2016).

How Imagine MyPath Develops Students' Algebraic Thinking

Imagine MyPath takes a problem-solving approach to help students learn to think and reason algebraically. Throughout the curriculum, there is an intentional focus on generalized **arithmetic**. Lessons encourage students to apply their understanding of the **properties of operations** when solving problems. For instance, in a Grades K–5 lesson, students learn about the commutative property as they explore patterns on a multiplication table (Figure 149). When finding the multiples of three, Imagine MyPath uses an array to help students see how 3 × 5 = 15 is equivalent to 5 × 3 = 15. This illustrates how the factors can be multiplied in any order (and the result will remain the same) and reinforces students' understanding of this property. In upper grade levels, students apply their understanding of properties when solving more challenging problems, like linear equations with one variable (Figure 150). In this lesson, the teacher guides students through the process of isolating the variable to help address potential misconceptions. Scaffolding is used to break down the problem into manageable steps to help students understand what property is being used at each step in the process (Figure 151).

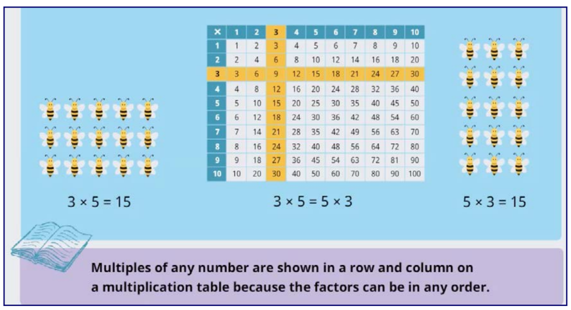


Figure 149. *Grades K–5 students analyze patterns on a multiplication table.*

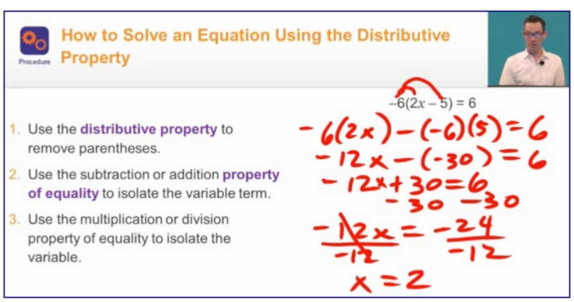


Figure 150. *Grades 6–12 students apply properties of operations.*

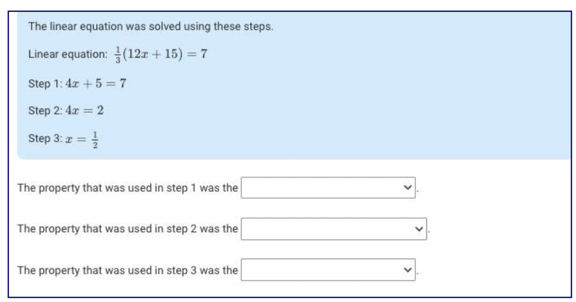


Figure 151. *Lessons break down a problem into manageable steps.*

Students explore the meaning of a **variable** as they practice basic algebraic manipulations with equations using a range of models. For instance, Grades K–5 lessons present equations with unknown quantities in different positions (Figure 152). Different symbols are also used to represent unknown or varying quantities (e.g., blanks, letters, question marks). As students solve problems in Grades 6–12, explicit instruction is used to teach students how to model, write, and solve one-step variable equations. In Figure 153, the teacher uses a table to help organize the steps for solving a word problem. First, they use the table to identify the type of mathematical situation, followed by the operation needed to solve it. Then, they model how to translate the problem into a mathematical equation. As students progress in their ILP, they solve problems in which they write and graph linear equations with two or more variables. As students solve linear equations and inequalities with multiple variables, lessons incorporate a variety of representations (e.g., ordered pairs derived from tables, algebraic rules, graphs) to promote connections across concepts (Figure 154).

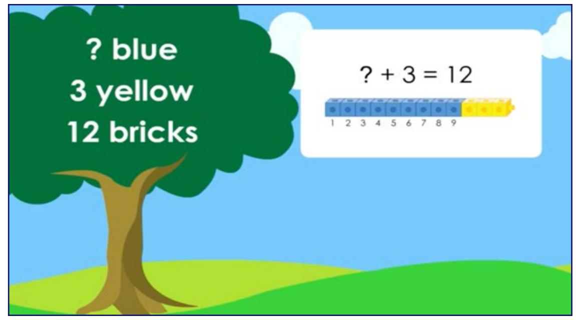


Figure 152. *Grades K–5 students solve equations with an unknown quantity.*

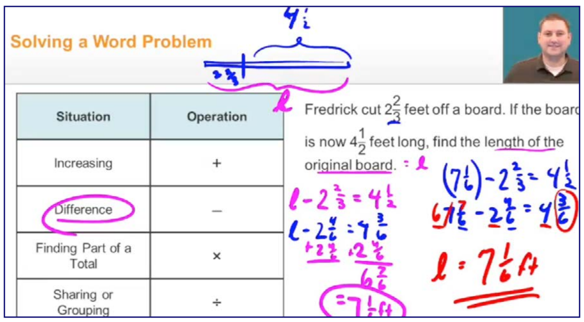


Figure 153. *Grades 6–12 students solve one-step equations with variables.*

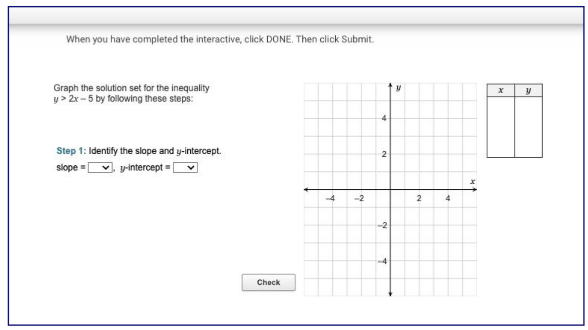


Figure 154. *Grades 6–12 students graph inequalities.*

Across the curriculum, Imagine MyPath supports students' relational understanding of **equivalence**. Lessons strategically present number sentences from left to right (a + b = c) and right to left (c = b + a) to minimize students' tendency to view the equal sign as an action when they solve problems (Figure 155). Lessons incorporate visuals, like a balance scale, to represent the meaning of equivalence. These lessons help students make associations between a balance scale and a balanced equation (Figure 156). This prompts students to recognize the relationship between procedures and concepts used in arithmetic and those used in algebra. In a Grades 6–12 lesson, students use interactive algebra tiles to model the expression and help develop a conceptual understanding (Figure 157). They also apply their understanding of the distributive property to create equivalent expressions.

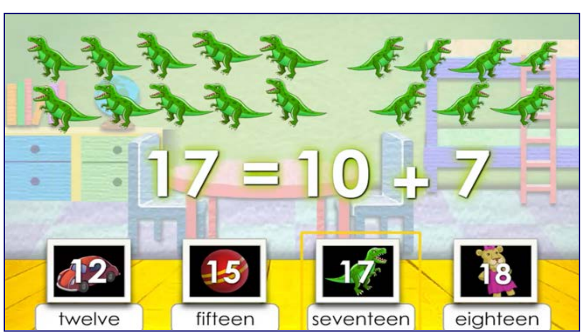


Figure 155. *Grades K–5 lessons promote a relational understanding of equivalence.*

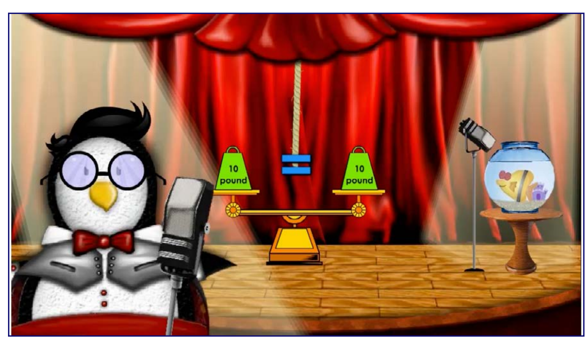


Figure 156. *Grades K–5 use balance scales to explore the equal sign.*

Imagine MyPath Grades 6–12 lessons provide ample opportunities for students to learn how to simplify and solve **linear equations, quadratic equations,** and **inequalities**. For instance, students make sense of and solve problems situated in the real world (Figure 158). Recognizing that many students struggle with concepts containing the negative sign, lessons also provide explicit instruction and scaffolding to model the process when solving inequalities with negative coefficients using a number line (Figure 159), an effective strategy for building conceptual understanding.

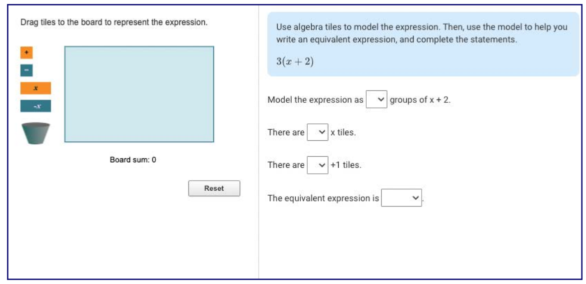


Figure 157. *Grades 6–12 students use interactive algebra tiles.*

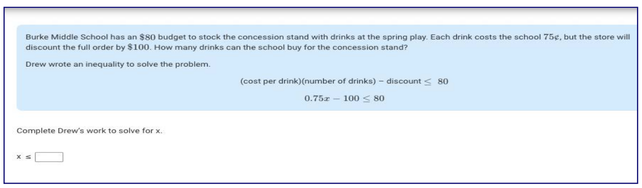


Figure 158. *Grades 6–12 students solve real-world problems.*

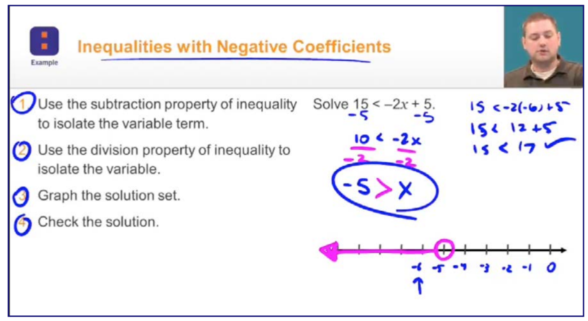


Figure 159. *Grades 6–12 students solve inequalities with negative coefficients.*

Exploring different types of **patterns** can support students' use of additive and multiplicative reasoning skills. In a Grades K–5 lesson, students are introduced to an example of a pattern (e.g., tile mosaics) and asked to determine what numbers will come next in the pattern (Figure 160). The teacher provides a visual model to help students determine the rule for this pattern (add three). This type of pattern reinforces additive thinking; however, as students begin to reason multiplicatively, patterns that have an underlying multiplicative structure help promote their attention to the covarying quantities. Students engage in multiplicative thinking as they analyze the relationship between quantities in the table and graph the numerical pattern on a coordinate plane (Figure 161). Students practice writing algebraic equations to represent the pattern rule (e.g., y = x + 5) and fill in a table with the input and output values, which simultaneously helps them learn to generalize.

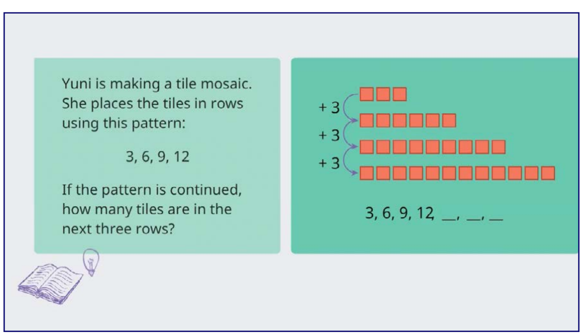


Figure 160. *Grades K–5 students explore repeating patterns.*

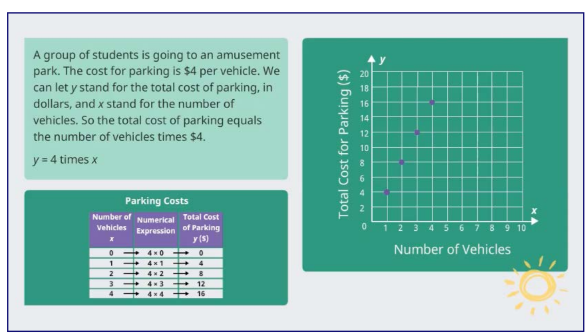


Figure 161. *Grades K–5 students explore covarying relationships.*

Given its applicability to students' daily lives, Imagine MyPath places a strong emphasis on **functions**. Students are taught to identify and describe functions in real-world scenarios, tables, graphs, and equations. In Figure 162, students explore real-world relationships and determine whether it is a function. In this lesson, the teacher explains why the scenario is not a function by highlighting the gap in the graph and discussing how the data display students of the same age with different heights. In another example, students learn to interpret rate of change from a graph and a table to differentiate functions as linear or nonlinear (Figure 163). Students are reminded, "The rate of change of a graph is found by determining how much the function is rising or falling each time it runs." In another Grades 6–12 lesson, students extend their understanding of functions by determining whether the definition of a function also applies to a quadratic function. The teacher presents a problem involving a medieval catapult (Figure 164). They explicitly model how to read the data in the table and graph the quadratic function. They explain, "As time increases, my height increases until it reaches the maximum height. It then decreases the same amount but time continues to increase." To reinforce understanding, students analyze graphs and compare the rate of change of quadratic, linear, and exponential functions (Figure 165).

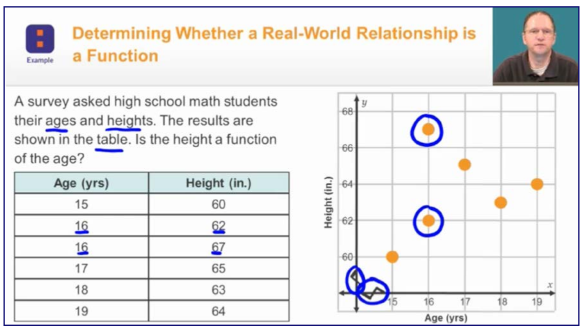


Figure 162. *Grades 6–12 lessons use real-world contexts to teach functions.*

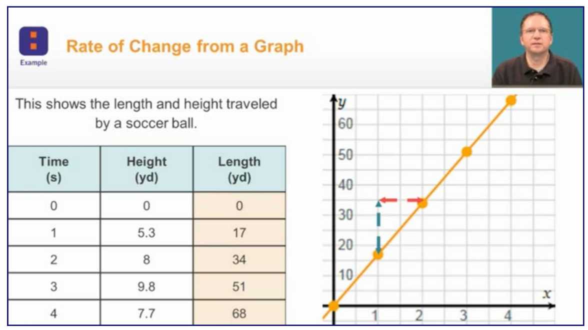


Figure 163. *Grades 6–12 students interpret the rate of change.*

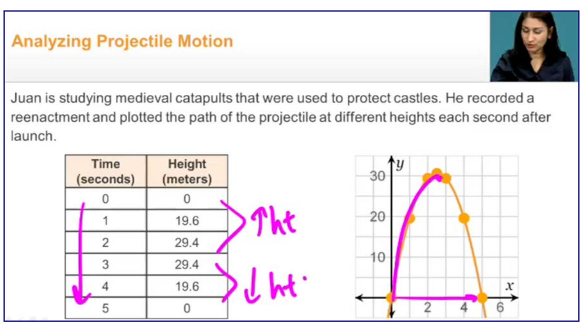


Figure 164. *Grades 6–12 lesson model how to graph a quadratic function.*

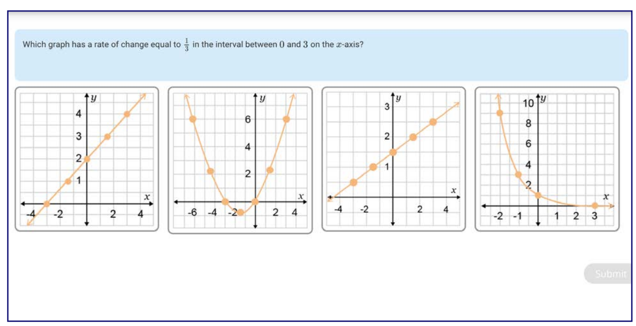


Figure 165. *Grades 6–12 students compare rate of change.*

Measurement and Data

The concept of measurement permeates students' everyday lives, which highlights its relevance and practicality in mathematics curricula. Yet, international comparisons reveal students' performance on measurement tasks in the United States remains low, and instruction often emphasizes low-level concepts and skills (Gavin et al., 2013). Understanding measurement is not only important for daily life, but also for its strong connections to other mathematical concepts, like rational numbers (Brendefuret al., 2013) and geometry (Clements et al., 2021; Reinhold et al., 2020). To prepare students to be internationally competitive, instruction should promote a conceptual understanding of measurement concepts and processes (e.g., measurement conversion, length, weight, angles, perimeter, area, surface area, volume). Instruction should also provide relevant problem contexts, encourage students' use of models and manipulatives, and intentionally address misconceptions (Gavin et al., 2013; Seah & Horen, 2020; Tan-Sisman & Aksu, 2016).

How Imagine MyPath Supports Students' Understanding of Measurement

Lessons integrate real-world scenarios, models, representations, and video demonstrations to enhance students' conceptual understanding of **measurement**. In a Grades K–5 lesson, students learn about length (Figure 166). The teacher models how to measure the length of a ribbon (represented as a horizontal line) using squares of the same size. After lining up 18 squares to match the ribbon's length, the teacher counts them from left to right. The teacher emphasizes how precise measurement cannot have any gaps or overlap, a common misunderstanding for students.

Recognizing that perimeter and area can be confusing, Imagine MyPath lessons are designed to help students understand the mathematics underlying these concepts and how these concepts can apply to their lives. In a Grades K–5 lesson, students learn how to use tiles to measure the area of a rectangle (Figure 167). Students learn how to use different strategies (counting each row of tiles, skip counting, and multiplying the side lengths) to determine the area. This is directly connected to multiplication and students' prior experiences solving problems using arrays. It also prepares them to solve problems involving surface area and volume, like the one displayed in Figure 168. In this Grades 6–12 lesson, students learn how to find the area of the faces of a 3-dimensional figure. Instruction models how to use nets (unfolded image of a 3-dimensional figure) to find the area of a triangular prism before students practice finding the total surface area of prisms and cylinders using formulas.

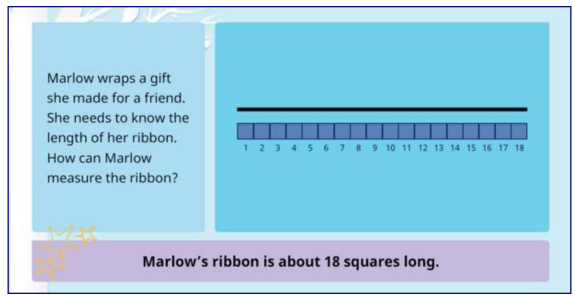


Figure 166. *Grades K–5 students learn how to measure length.*

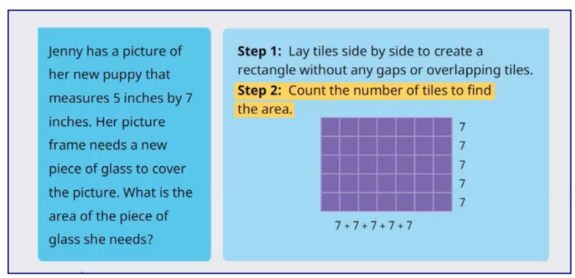


Figure 167. *Grades K–5 students use square tiles to measure area.*

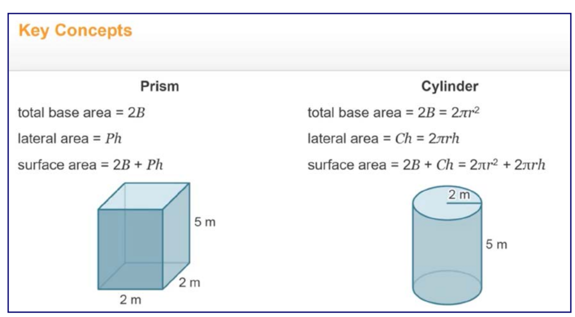


Figure 168. *Grades 6–12 students solve problems involving surface area.*

Students learn to apply formulas to find the circumference of a circle, the area of a circle, and the volume of a cylinder, pyramid, and cone. In a Grades 6–12 lesson, students approximate the circumference of a circle given the diameter or radius using 3.14 (π). The teacher begins by introducing key vocabulary words (Figure 169) and then outlines the process to find the diameter and radius of a circle. Students are explicitly taught to write down the appropriate formula, fill in the unknown values, and then solve for the unknown values. Breaking down the process into steps helps students understand the process for finding the diameter and radius, while simultaneously preparing them to solve more complex concepts involving the radian measure of a central angle and arc length (Figure 170).

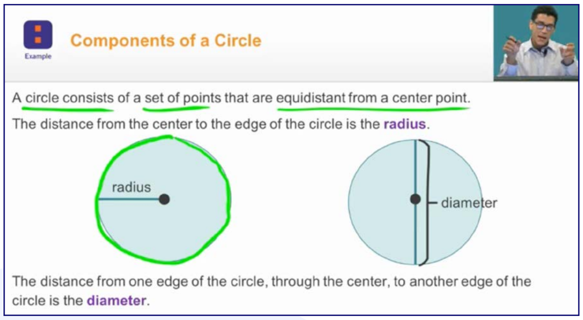


Figure 169. *Grades 6–12 teachers introduce key vocabulary.*

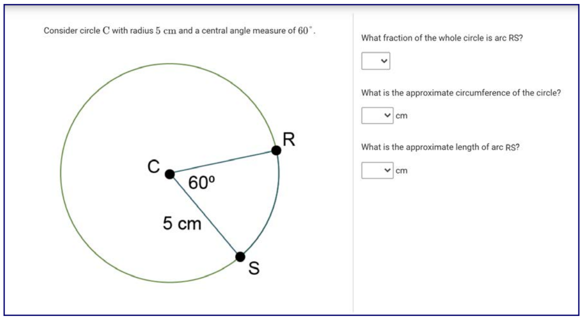


Figure 170. *Grades 6–12 students solve problems involving arc length.*

As the demand for statistical literacy grows, opportunities to reason about data, statistics, and probability have assumed a much deeper and wider role in mathematics curricula (Bargagliotti et al., 2020; Groth, 2013; Leavy et al., 2018). **Statistical reasoning** involves interpreting real data sets, graphic representations, and statistical summaries (Garfield, 2002). It also includes concepts like distribution, sampling, measures of center, measures of variability, probability, and inferences. While the goal is to understand the interaction between data and context, research has found many students struggle to make sense of these concepts (Bryant & Nunes, 2012; Groth, 2013; Rahmi et al., 2021) and are unable to develop reasoning skills at the level that is needed (Glancy et al., 2017). A growing body of research recommends the following instructional practices to expand students' statistical reasoning skills:

- Provide real and motivating data sets; organize and display data using different representations (e.g., tables, charts, graphs); explore statistical concepts beyond measures of center (e.g., variability, inferences); and employ different statistical tests (Biehler et al., 2013; Groth, 2013).
- Utilize technology to make statistics visual, interactive, and dynamic, as this helps emphasize concepts over computation and offers engaging opportunities to analyze data (Biehler et al., 2013).

How Imagine MyPath Develops Students' Statistical-Reasoning Skills

Imagine MyPath helps students develop **statistical reasoning** skills from an early age, by providing interactive lessons that contain real and engaging data. In Grades K–5, students are exposed to a variety of representations (e.g., tables, graphs, charts), learn how to read and draw conclusions from picture and bar graphs (Figure 171), and answer questions about the data (Figure 172). Students learn to read the data and attend to features in the representations, such as how to use the key to understand data in the picture graph or attend to the x-axis and y-axis labels on a bar graph.

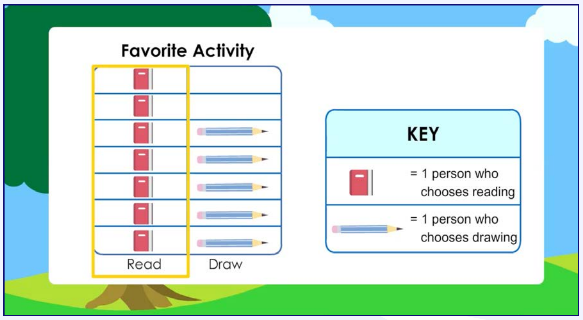


Figure 171. *Grades K–5 students analyze data in a picture graph.*

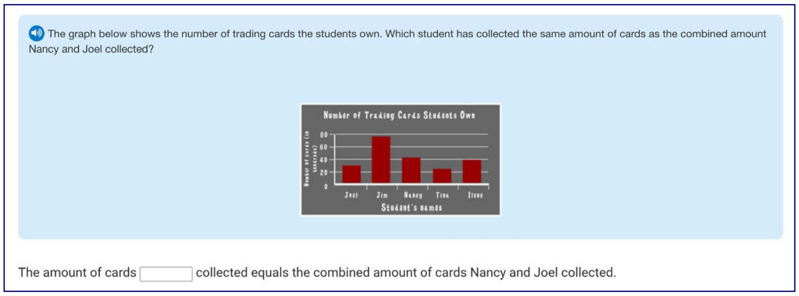


Figure 172. *Grades K–5 students answer questions about data in a bar graph.*

Lessons also incorporate a range of concepts for students to explore, including measures of central tendency, variability, sampling, probability, and inferences. Real-world situations are used to encourage students to apply their thinking and make sense of concepts in context, beyond simply viewing data as numbers. In a Grades 6–12 lesson, students learn how to construct a circle graph and interpret data to make predictions (Figure 173). In another lesson, students practice finding the probability of an event, expressing it as a ratio, and using it to make predictions about real-life events. This lesson capitalizes on the use of technology to make statistics visual, interactive, and dynamic. Students use a spinner to conduct an experience, collect and organize data in a table, and make predictions about a color's experimental probability (Figure 174). Students also practice analyzing and translating data in tables and graphs into equations (e.g., P(P) = 4/12 = 1/3).

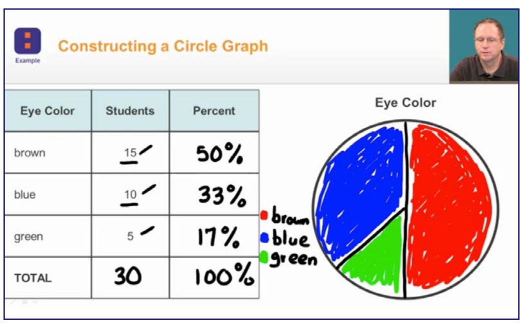


Figure 173. *Grades 6–12 teachers model how to construct a circle graph.*

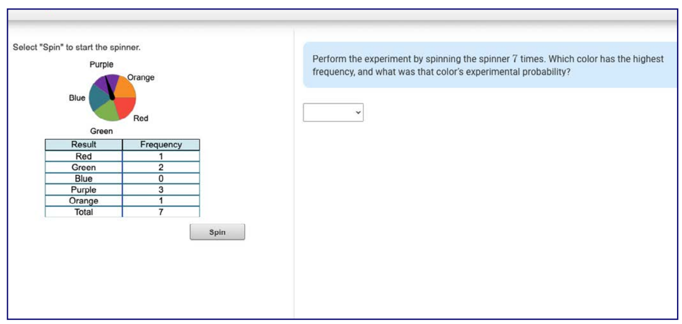


Figure 174. *Grades 6–12 students analyze data and make predictions.*

Geometry

Geometric thinking is described as the thought processes students employ as they learn geometry and do geometry (Groth, 2013). Students' early experiences in **geometry** lay the foundation for their work in later grade levels (Biber et al., 2013; Clements & Sarama, 2021; Dindyal, 2015; Moss et al., 2015). For instance, students' conceptual understanding of 2-dimensional and 3-dimensional shapes and the relationships between them (Van de Walle et al., 2018a) provide a foundation for their work in the middle and secondary grades (e.g., expressing theorems algebraically, geometric measurement, congruence, proof) (Groth, 2013). The van Hiele Model of Geometric Thinking catalogues students' progressive understanding of geometric reasoning into five levels (van Hiele, 1986):

- Level 1 (Visualization)—identifies geometric shapes, but does not focus on properties or attributes
- Level 2 (Analysis)—recognizes shapes have different properties, and can identify shapes by that property, but does not recognize the relationship between properties

- Level 3 (Informal Deduction)—recognizes and describes the relationships between objects and shapes, and engages in "if…then" reasoning
- Level 4 (Formal Deductive)—constructs proofs, analyzes informal arguments and the structure of a system, and begins to establish geometric truth based on logic
- Level 5 (Rigor)—understands abstract geometry and sees the "construction" of geometric systems

Despite the importance of this domain, geometry typically receives less attention than other domains (Clements & Sarama, 2021) and instruction often emphasizes vocabulary over application or concept development (Geddes & Fortunato, 1993; Sinclair & Bruce, 2015). To strengthen students' geometric thinking and foster connections between geometry and other mathematics domains, instruction should:

- Provide opportunities for students to reason about 2- and 3-dimensional shape attributes and properties using precise language, decompose shapes, compare examples and non-examples, and make connections between concepts and the real world (Clements & Sarama, 2021; Dobbins et al., 2014; Groth, 2013; Resnick et al., 2020; Seah & Horne, 2020).
- Integrate the use of dynamic technology to explore the visual nature of geometry (e.g., interactive manipulatives or geoboards) (Chan & Leung, 2014; NCTM, 2014; Sinclair & Bruce, 2015) and concepts like geometric measurement and transformations (Groth, 2013).
- Draw on van Hiele's Model of Geometric Thinking to support students' progression through the five levels (Breyfogle & Lynch, 2010; Groth, 2013).

How Imagine MyPath Promotes Students' Geometric Thinking

Imagine MyPath knows that students develop stronger connections to content when they can apply their understanding to the real world. In Grades K–5 geometry lessons, students practice identifying shapes in familiar settings, such as the park (Figure 175). Lessons reinforce vocabulary and introduce students to shape properties and attributes. For instance, all triangles have three sides and three corners. In a Grades 6–12 lesson, students learn how to classify angles (right, acute, obtuse) and lines (parallel, perpendicular) in 2-dimensional figures. Key vocabulary is defined on screen (Figure 176) and students are encouraged to look up these words in the glossary. Lessons also integrate various real-world images (e.g., railroad tracks) to help students recognize the relevance of geometry in the world around them.

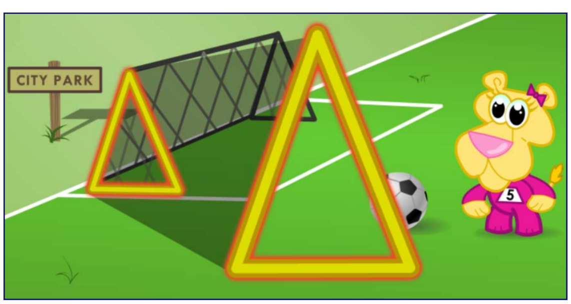


Figure 175. *Grades K–5 students identify shapes in familiar settings.*

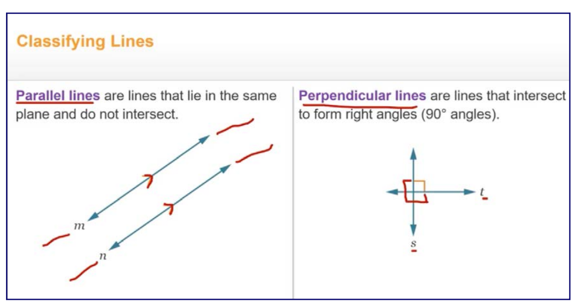


Figure 176. *Grades 6–12 students classify lines in 2-D figures.*

Students practice composing and decomposing shapes using interactive manipulatives to develop their geometric thinking skills (Figure 177). Students also learn how to model 2-dimensional shapes on a geoboard, analyze and compare those shapes, and investigate the relationship between shape attributes (e.g., sides and corners) (Figure 178). In a Grades 6–12 lesson, students explore transformations (translations, reflections, rotation, and dilations) on a coordinate plane (Figure 179).

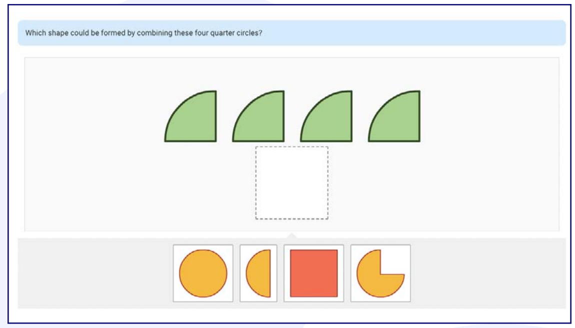


Figure 177. *Grades K–5 students compose and decompose shapes.*

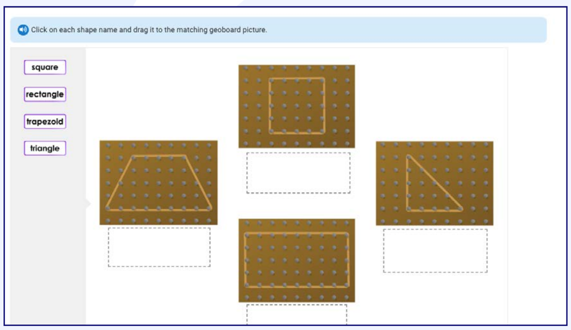


Figure 178. *Grades K–5 students model 2-D shapes using a geoboard.*

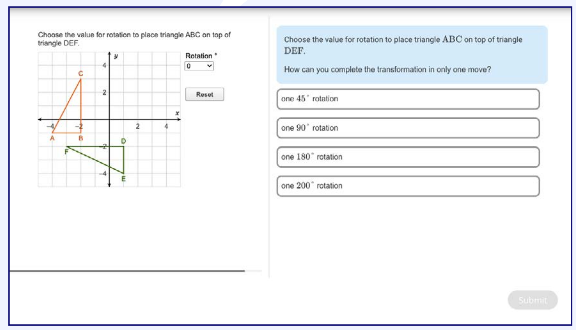


Figure 179. *Grades 6–12 students explore transformations on a coordinate plane.*

Imagine MyPath lessons consider all five levels included in van Hiele's Model of Geometric Thinking. Lessons provide opportunities for students to visualize, analyze, reason deductively, and engage in rigorous thinking. For instance, in a Grades K–5 lesson, students practice identifying faces, edges, and vertices of a cube (Figure 180). According to van Hiele's Model of Geometric Thinking, this lesson would be categorized as a Level 2 because of its emphasis on recognizing and describing different shape properties. In a more advanced geometry lesson, categorized as a Level 4, students construct proofs and use deductive reasoning to craft informal arguments. Explicit instruction is used to teach different proof formats and how to prove a statement with justifications (e.g., theorems, proofs). Students use the interactive two-column proof tool to identify statements and reasons they are able to make those deductions (Figure 181). This helps students appreciate the meaning and importance of deduction.

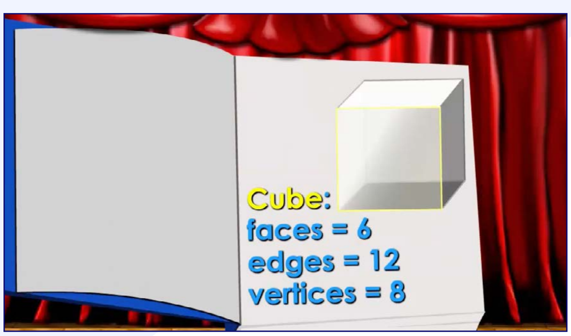


Figure 180. *Grades K–5 students identify shape attributes and properties.*

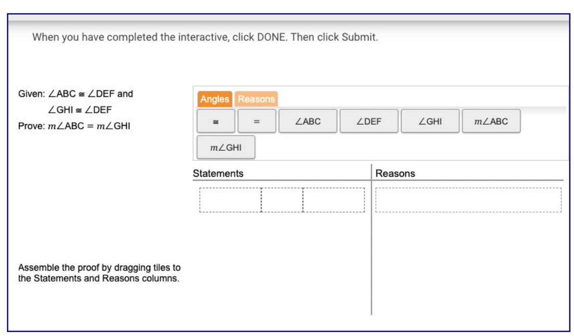


Figure 181. *Grades 6–12 students construct proofs and use deductive reasoning.*

Principle 5. Deliver Actionable Data to Inform Instructional Decision Making

There is widespread agreement that **data-based decision making** can strengthen instruction and improve student achievement (Lai et al., 2014; Poortman & Schildkamp, 2016; van Geel et al., 2016). Progress monitoring and achievement data enable educators to assess students' responsiveness to instruction and make important adaptations to meet the needs of their students (Fuchs & Fuchs, 2015, 2016; Gersten, Compton, et al., 2009; National Center for Learning Disabilities, n.d.; NRC, 2012; Stoiber & Gettinger, 2015). Collecting multiple sources of data over time helps educators prioritize the skills needed to improve learning (Schildkamp, 2019). When they have a clear purpose for data, as well as the ability to make sense of what the data mean, teachers can more effectively "set appropriate student learning goals; monitor and check to see if students are reaching their goals; and support students in developing the ability to monitor and check their own goal attainment" (Lai & Schildkamp, 2013, p. 15). By focusing on data as a tool for continuous improvement, teachers can use actionable insights to strengthen instruction and enhance student learning (Mandinach & Schildkamp, 2021).

How Imagine MyPath Capitalizes on Actionable Data to Inform Instructional Decision Making

Benchmark and formative assessment data enable teachers to measure students' strengths and areas of growth, make informed decisions about instruction, and supplement students' ILPs as needed. Imagine MyPath's priority mapping and adaptive system supports teachers in an increasingly time-intensive, individualized, and data-driven world. Teachers and administrators gain actionable insights on student progress and engagement with various reporting tools, empowering them to make data-driven decisions to personalize instruction and intensify intervention.

- ***Imagine MyPath Assessments* (Benchmark assessments):*** The Imagine MyPath Assessments are fully adaptive assessments that measure skills in Grades K–12 reading/English language arts and mathematics. The MyPath Assessment for reading/English language arts measures student skills in four domains for Grades K–12, including foundational skills, language, literary analysis, and informational-text analysis. The MyPath Assessment for mathematics measures student skills in four domains for Grades K–8, including number sense, operations and algebraic thinking, geometry and measurement, and data analysis. For high school, the MyPath Assessment for mathematics measures five domains, including number and quantity, algebra, functions, geometry, and statistics and probability. Information generated from these assessments supports data-based decision making at the classroom, school, and district level.

Each assessment includes an introduction and three testing parts (with rest breaks in between), designed to be completed in one class period (approximately 45–60 minutes). The introduction includes an interactive and age-appropriate activity that primes students to take the test. Then, students complete

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**Imagine MyPath can also integrate data from NWEA® MAP® Growth, Renaissance Star® or teachers' input to set and adjust ILPs, should schools choose not to administer the Imagine MyPath Assessments.*

three adaptive parts that present a set of multiple-choice and technology-enhanced items targeting their current skill level (Figures 182 and 183). Students respond to approximately 30 to 45 items across all three stages, with test length varying across grades and for individual students. This assessment determines the ability and assessed grade level at regular intervals during the year, regardless of their actual grade. Imagine MyPath also provides a grade-level placement and a learning path, which teachers can review. Teachers can reassign the assessment or adjust the grade-level placement, if needed, to ensure the best fit for each student.

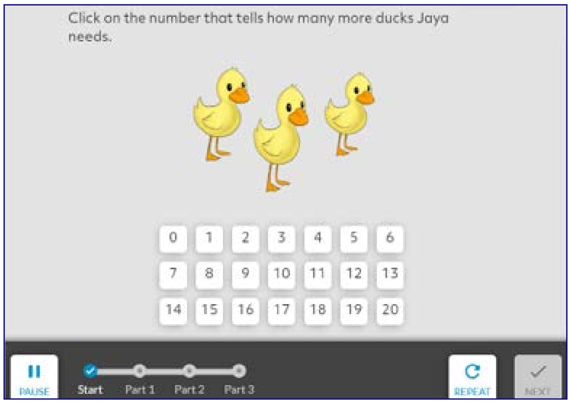

Figure 182. *Sample Imagine MyPath Assessment mathematics question.*

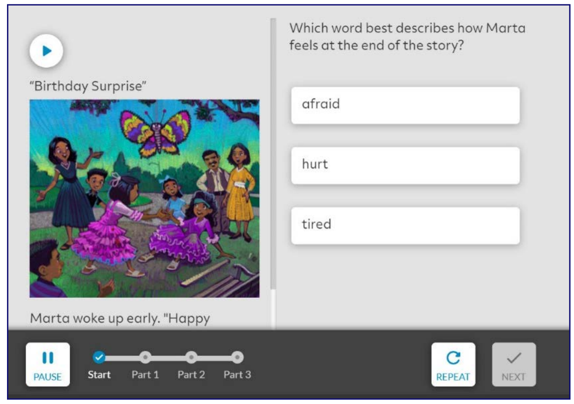

Figure 183. *Sample Imagine MyPath Assessment reading question.*

- ***Formative assessments:*** Imagine MyPath monitors and adapts learning through ongoing Mastery Checks that assess students' knowledge of the content within and across lessons (Figures 184 and 185). These formative assessments drive the adaptivity of the curriculum by targeting skill gaps as they emerge and allowing students to skip content they have already mastered. There are three five-item assessments within each lesson (there are two assessments in the *Early Literacy Bundles* and reading foundations lessons). Mastery Checks pull from a pool of items, so each Mastery Check is different. If a student answers at least four out of the five items correctly (or 80%) on the first or second Mastery Check, they test out of the lesson. Teachers can see students' progress at the domain and skill level in real time, allowing them to intervene as needed.

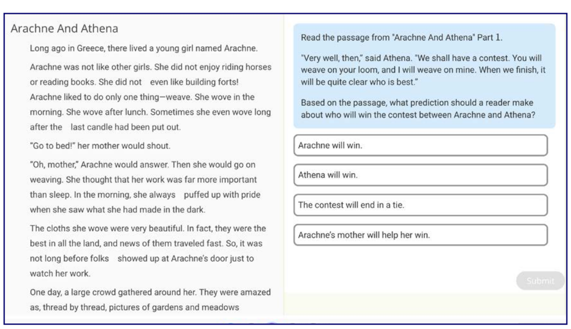

Figure 184. *Sample reading Mastery Check item.*

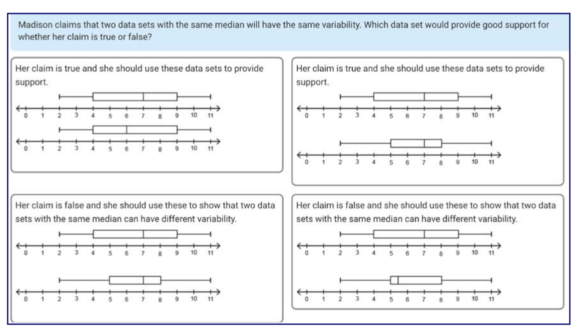

Figure 185. *Sample mathematics Mastery Check item.*

The program includes a range of educator-facing reports to assess progress and implementation fidelity. Each of these reports can be viewed at the classroom, school, or district level so teachers and administrators can evaluate progress at various levels.

- ***Class summary dashboard:*** This user-friendly interface provides an overview of students' progress on content mastery ("students to focus on" and "lessons to focus on") and usage (implementation fidelity) (Figure 186). Teachers can click on individual students to see their progress along their ILP, such as the number of lessons completed, number of Mastery Checks passed and failed, or number of lesson attempts. Based on a student's progress, teachers can modify a student's ILP by adding, removing, or resetting lessons. For example, if the teacher clicks on Jesus's progress report (Figure 187), they can see he struggles with multiplication (indicated by multiple failed lessons). The teacher can reference a specific lesson to see his number of attempts, answers on each of the Mastery Check items, and whether he would benefit from reteaching opportunities (after three failed attempts). Using Imagine MyPath's printable resources, the teacher can use these data to inform instruction by working with Jesus individually, grouping students with similar difficulties, or reviewing multiplication with the whole class.

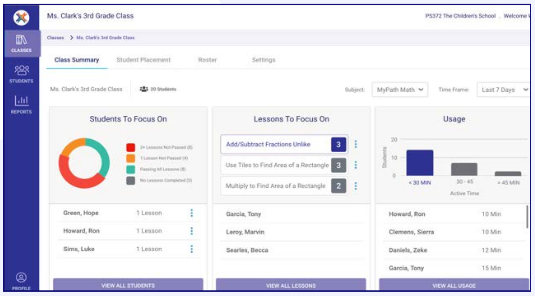

Figure 186. *Teacher dashboard provides overview of students' progress.*

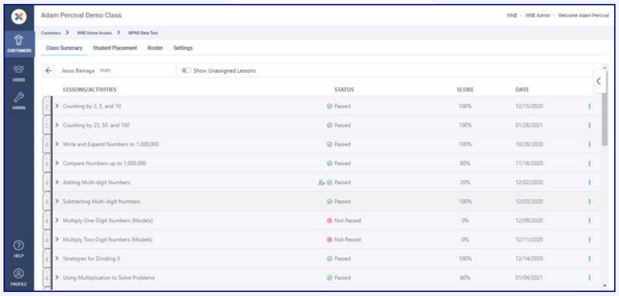

Figure 187. *Teachers can view student's progress on their ILP.*

- ***Class progress report:*** This report provides a quick view into how the class is doing and which students need help and on what lessons. There are two different views for this report—Domain Overview and Lesson Overview. The Domain Overview is the default view and displays how well each student in the class has performed in each domain (Figure 188). This is determined by the completed lessons for each domain.

- ***Imagine MyPath Assessment report:*** Teachers and administrators can see detailed information about student performance on the Imagine MyPath Assessment, as well as summaries at the class, school, and district level. Imagine MyPath Assessment data include a scaled score, assessed grade, and grade classification (above grade level, on grade level, one grade below, two or more grades below) (Figure 189).

- ***Placement report:*** Teachers and administrators can see an overview of students' grade placement and whether the placement is based on a benchmark-assessment result or educator judgment. A student's placement level can be updated with each benchmark assessment.

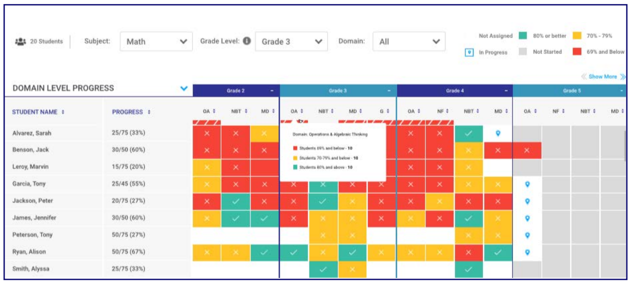

Figure 188. *Class progress report provides a Domain Overview.*

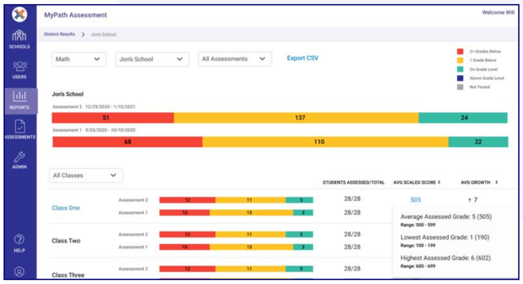

Figure 189. *Imagine MyPath Assessment report.*

Principle 6. Optimize Student Motivation and Engagement

Students' beliefs and perceptions about their academic abilities influence their **motivation** to learn. For instance, students who value what they are learning and believe they are capable of successfully completing a task are more likely to exert greater effort, demonstrate persistence, and show higher academic achievement (Chiu, 2017; Ryan & Deci, 2000). Research shows academic self-efficacy, positive self-concept, and motivation can positively affect reading achievement (Lee & Jonson-Reid, 2016; Parker et al., 2014; Yang et al., 2018) and mathematics achievement (Arens et al., 2020; Gunderson et al., 2018; Holenstein et al., 2021; Schöbera et al., 2018). Self-efficacy is defined as a student's belief in their ability to successfully complete a task or achieve an academic goal (Bandura, 1997; Pajares, 1996), whereas self-concept refers to a student's self-perceived competence related to a general domain (e.g., mathematics, reading) (Marsh & Martin, 2011). Students' beliefs and perceptions are important because these can influence, what, when, and how they choose to learn (Hartnett, 2016; Schunk & Usher, 2012). Given the relationship between these constructs, it is critical that educators thoughtfully consider how to motivate students to learn. Research recommends actively addressing the following to improve student motivation, engagement, and academic achievement.

- Model self-monitoring and goal-setting behavior (Harris et al., 2015; Wang et al., 2019). Provide opportunities for students to set and monitor their own goals, progress, and overall performance (Bai et al., 2020; Gnauk et al., 2012).
- Incorporate frequent feedback that encourages effort, emphasizes mastery and persistence with challenging tasks, and promotes positive mindsets (Liao et al., 2019; Margolis & McCabe, 2006).
- Promote behavioral engagement (actions students take during learning) and psychological engagement (processing required to learn) to enhance learner outcomes (Clark & Mayer, 2016).
- Integrate gamified motivational elements such as points, badges, avatars, and progress reports to provide continued feedback to enhance motivation and engagement (Alshammari, 2020; Bai et al., 2020; Birk et al., 2016; Filsecker & Hickey, 2014).

How Imagine MyPath Optimizes Student Motivation and Engagement

Imagine MyPath promotes self-efficacy and positive self-concept with engaging, interactive learning environments that reward students for their progress and **motivate** them to persist through challenging content. The lessons are built around a gradual-release instructional model to help students develop confidence in their abilities to succeed. Imagine MyPath models self-monitoring and goal-setting behavior. For example, in a reading lesson, students are asked questions such as, "Do you see any evidence?" and "Do you see any clues?" This encourages them to actively monitor their reading comprehension. Metacognitive bubbles also appear during Guided Practice to remind students to monitor their comprehension. Frequent progress monitoring (i.e., Mastery Checks within lessons) allows students to assess their learning goal progress. Students can also visually track their progress on a map within the ILP home screen as they work toward mastering grade-level content (Figures 190 and 191).

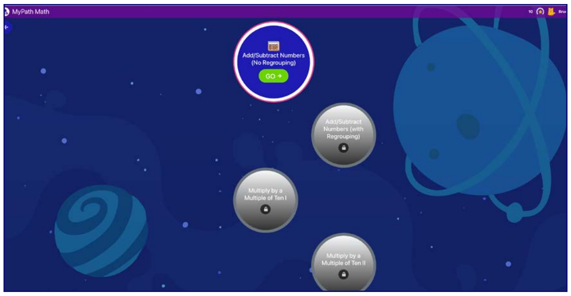

Figure 190. *Grades K–5 students' home screen.*

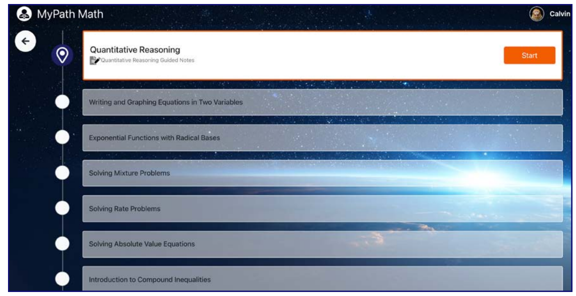

Figure 191. *Grades 6–12 students' home screen.*

Imagine MyPath recognizes the importance of providing frequent and focused feedback to improve **motivation**. The program incorporates encouraging and celebratory prompts to motivate students, such as "Ready to show us what you can do?" and "Almost had it!". Even when students do not pass a Mastery Check, they receive positive reinforcement for attempting the challenge, such as "Nice effort! Let's get some more practice. You'll be an expert in no time." They can also earn points. They can use their points to unlock customization features within the program (e.g., backgrounds, sidekicks, avatars). If a student fails more than three lessons, their ILP recalibrates to minimize frustration and ensure they experience success.

The curriculum provides age-appropriate presentations to maintain student engagement and **motivation**. Lessons promote behavioral engagement using various multimedia presentation formats (e.g., engaging videos, Figures 192 and 193), teaching styles (e.g., explicit instruction, Figure 194), and response options (e.g., drag-and-drop responses, Figure 195). To promote psychological engagement, lessons monitor student progress and provide targeted instruction to address learning gaps. Lessons activate students' prior knowledge, provide strategy instruction, and encourage self-explanation of material. This helps support the cognitive learning process (e.g., connecting already-learned content to new content, concept mapping), which, in turn, strengthens the program's effectiveness.

Figure 192. *Character dialogue fosters student engagement.*

Figure 193. *Imagine MyPath lessons integrate engaging videos.*

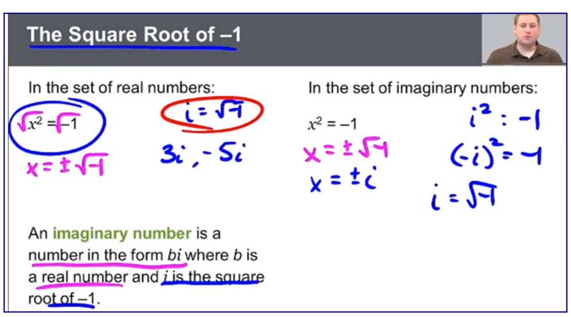

Figure 194. *Imagine MyPath's instructional approach engages students.*

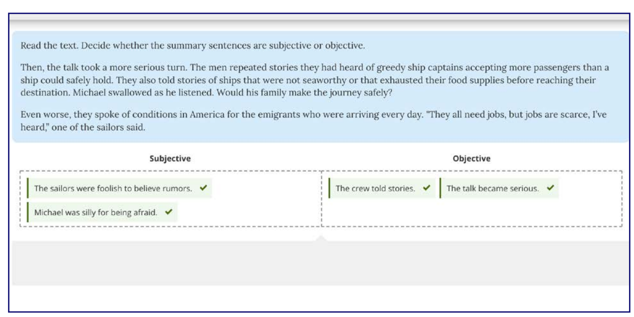

Figure 195. *Imagine MyPath lessons provide multiple response options.*

Figure 196. *Students earn points to customize their dashboard.*

Throughout the program, students earn points, badges, rewards, sidekicks, and avatars. Students earn points (stars) to customize features on their dashboard (Figure 196). In Grades K–8, students can trade in points for new sidekicks (Figure 197) and backgrounds; in Grades 9–12, students can trade in their points to unlock new avatars and backgrounds (Figure 198). The more points they earn, the more customization features students can choose from.

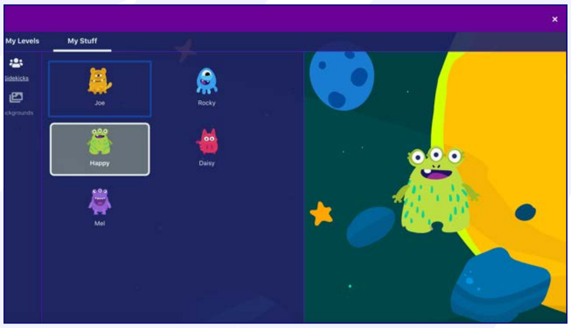

Figure 197. *Grades K–8 students trade points for new sidekicks.*

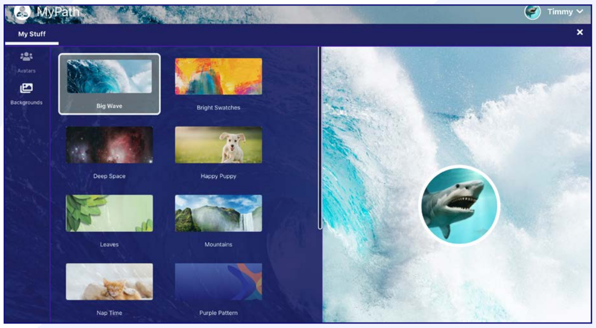

Figure 198. *Grades 9–12 students trade points to unlock new backgrounds.*

Conclusion

Imagine MyPath is a next-generation learning environment designed for students in Grades K–12. The curriculum integrates research-based reading and mathematics instructional practices to maximize student learning. All students receive accessible, explicit, age-appropriate instruction, no matter their grade level. Smart Sequencer™ technology is used to prioritize content and continuously adapt instruction based on student performance, providing every student with a pathway to grade-level success. Reading prioritizes reading foundations (word-recognition skills) and comprehension (i.e., language-comprehension skills), with a strong focus on reading comprehension across literary and informational texts. In mathematics, lessons focus on building students' conceptual understanding of number and operations, algebra, measurement and data, and geometry. Throughout the curriculum, lessons are grounded in real-world contexts to increase engagement and help students make connections to their daily lives. Teacher data dashboards offer actionable insights on student progress, optimizing data-based decision making. Student dashboards, rewards, positive behavioral support, and customizable features motivate students to track their progress and take charge of their learning. Imagine MyPath has revolutionized the student experience and become a powerful tool for empowering educators and helping all students reach their full potential.

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